Math Analysis Practice

Name: Key

*Leave answers in simplest radical/fractional form. NO DECIMAL ANSWERS!

I) Use the sum or difference identities.
Find the exact value of each by rewriting the angle as the sum or difference of 2 special angles.

1. $\cos\frac{\pi}{12}$

$\cos\left(\frac{4}{12}\pi - \frac{3}{12}\pi\right)$

$\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

2. $\sin 105°$

$\sin(60° + 45°)$

$\sin 60°\cos 45° + \cos 60°\sin 45°$

$\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

3. $\tan 15°$

$\tan(45° - 30°)$

$\frac{\tan 45° - \tan 30°}{1 + \tan 45°\tan 30°}$

$\frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\frac{\sqrt{3}}{3}} \cdot \frac{\left(1 - \frac{\sqrt{3}}{3}\right)\cdot 3}{\left(1 + \frac{\sqrt{3}}{3}\right)\cdot 3}$

$\boxed{\frac{3-\sqrt{3}}{3+\sqrt{3}}}$

Find the exact value of each expression by simplifying to a special angle.

4. $\cos 40°\cos 10° + \sin 40°\sin 10°$

$\cos(40° - 10°) = \cos 30° = \boxed{\frac{\sqrt{3}}{2}}$

5. $\sin\frac{\pi}{12}\cos\frac{7\pi}{12} - \cos\frac{\pi}{12}\sin\frac{7\pi}{12}$

$\sin\left(\frac{\pi}{12} - \frac{7}{12}\pi\right) = \sin\left(-\frac{6}{12}\pi\right) = \sin\left(-\frac{\pi}{2}\right) = \boxed{-1}$

6. $\frac{\tan 40° + \tan 20°}{1 - \tan 40°\tan 20°}$

$= \tan(40° + 20°)$

$\tan(60°) = \boxed{\sqrt{3}}$

II) Use the double or half angle identities. $2\left(\frac{1}{8}\pi\right)$

Find the exact value of each by rewriting the angle as half a special angle.

7. $\sin 195°$

$\sin\left(\frac{390°}{2}\right)$

$= -\sqrt{\frac{1-\cos 390°}{2}}$

$= -\sqrt{\frac{\left(1-\frac{\sqrt{3}}{2}\right)}{(2)^2}} = -\sqrt{\frac{2-\sqrt{3}}{4}}$

8. $\cos 165°$

$\cos\frac{330°}{2}$

$= -\sqrt{\frac{1+\cos 330°}{2}}$

$= -\sqrt{\frac{\left(1+\frac{\sqrt{3}}{2}\right)}{2}^2} = -\sqrt{\frac{2+\sqrt{3}}{4}}$

9. $\tan\frac{9\pi}{8}$

$\tan\frac{1}{2}\left(\frac{9}{4}\pi\right)$

$= \frac{1-\cos\frac{9}{4}\pi}{\sin\frac{9}{4}\pi}$

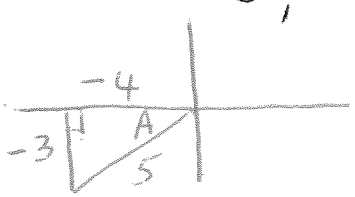
$\frac{\left(1-\frac{\sqrt{2}}{2}\right)^2}{\left(\frac{\sqrt{2}}{2}\right)^2} = \boxed{\frac{2-\sqrt{2}}{\sqrt{2}}}$

$\sqrt{2}-1$

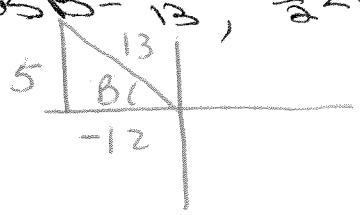
$\frac{2\sqrt{2}-2}{2}$

Set up a triangle, in the correct quadrant, for angle A and for angle B. Use the triangles to answer each question below.

$\sin A = \frac{3}{5}$, $\pi < A < \frac{3\pi}{2}$



$\cos B = \frac{-12}{13}$, $\frac{\pi}{2} < B < \pi$



10. Find $\sin(A-B)$

$\sin A\cos B - \cos A\sin B$

$\left(-\frac{3}{5}\right)\left(-\frac{12}{13}\right) - \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right)$

$\frac{36}{65} + \frac{20}{65} = \boxed{\frac{56}{65}}$

11. Find $\tan(A+B)$

$\frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\frac{\left[\frac{3}{4}+\left(\frac{-5}{12}\right)\right]12\cdot 4}{\left[1-\left(\frac{3}{4}\right)\left(\frac{-5}{12}\right)\right]12\cdot 4} \quad \frac{36-20}{48+15} = \boxed{\frac{16}{63}}$

12. Find $\cos 2B$

$\cos^2 B - \sin^2 B$

$\left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$\frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

13. Find $\cos\frac{A}{2}$

$\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}}$

$-\sqrt{\frac{1+\left(-\frac{4}{5}\right)}{2}\cdot\frac{5}{5}}$

$-\sqrt{\frac{5-4}{10}} = -\frac{1}{\sqrt{10}} = \boxed{-\frac{\sqrt{10}}{10}}$