Name: Ans Key

Secant and Tangent Lines

I. Find the equation of the secant line through the indicated points for each.

1). $f(x) = x^2 - 3x + 5$ from $x = 0$ to $x = 3$

$(0, f(0)) = (0, 5)$

$(3, f(3)) = (3, 5)$

$m = \frac{5-5}{3-0} = 0$

$y - 5 = 0(x - 0)$

$\boxed{y = 5}$

2). $f(x) = x^3 + 10$ from $x = -1$ to $x = 1$

$(-1, f(-1)) = (-1, 9)$

$(1, f(1)) = (1, 11)$

$m = \frac{11-9}{1-(-1)} = \frac{2}{2} = 1$

$\boxed{y - 9 = 1(x+1)}$

$y = x + 1 + 9$ or $\boxed{y = x + 10}$

3). $f(x) = x^4 + 10$ from $x = -1$ to $x = 1$

$(-1, f(-1)) = (-1, 11)$

$(1, f(1)) = (1, 11)$

$m = \frac{11-11}{1-(-1)} = 0$

$\boxed{y = 11}$

4). $f(x) = \sqrt{x-3} + 2$ from $x = 7$ to $x = 12$

$(7, f(7)) = (7, 4)$

$(12, f(12)) = (12, 5)$

$m = \frac{5-4}{12-7} = \frac{1}{5}$

$\boxed{y - 4 = \frac{1}{5}(x - 7)}$

or

$y = \frac{1}{5}x - \frac{7}{5} + \frac{20}{5}$

$\boxed{y = \frac{1}{5}x + \frac{13}{5}}$

5). $f(x) = \cos x$ from $x = 0$ to $x = \frac{3\pi}{2}$

$(0, \cos 0) = (0, 1)$

$(\frac{3}{2}\pi, \cos \frac{3}{2}\pi) = (\frac{3}{2}\pi, 0)$

$m = \frac{0-1}{\frac{3}{2}\pi - 0} = \frac{-1}{\frac{3}{2}\pi} = -\frac{2}{3\pi}$

$\boxed{y - 1 = -\frac{2}{3}\pi(x - 0)}$ or

$\boxed{y = -\frac{2}{3}\pi x + 1}$

6). Using the following table, find the equation of the secant line through $x = 10, x = 17$

x	-4	0	3	10	13	15	17
$f(x)$	-17	-2	13	19	11	28	34

$\boxed{y - 19 = \frac{15}{7}(x - 10)}$

$(10, 19)$

$(17, 34)$

$m = \frac{34-19}{17-10} = \frac{15}{7}$

use to find Instan. Rate of chang

$\boxed{\lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$

II. Find the instantaneous rate of change (derivative) for each at the given x value.

7). $f(x) = 3x - 1$; at $x = 0$

$\lim_{x \to 0} \frac{(3x-1) - (3(0)-1)}{x - 0}$

$\lim_{x \to 0} \frac{3x - 1 + 1}{x} = \lim_{x \to 0} \frac{3x}{x}$

$\lim_{x \to 0} 3$

$= \boxed{3}$

8). $f(x) = x^2$; at $x = 1$

$\lim_{x \to 1} \frac{x^2 - (1)^2}{x - 1}$

$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

$\lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$

$\lim_{x \to 1} x + 1 = 1 + 1 = \boxed{2}$

9). $f(x) = x^2 + x - 1$; at $x = 2$

$\lim_{x \to 2} \frac{(x^2 + x - 1) - (2^2 + 2 - 1)}{x - 2}$

$\lim_{x \to 2} \frac{x^2 + x - 1 - 5}{x - 2}$

$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}}$

$\lim_{x \to 2} x + 3$

$= 2 + 3 = \boxed{5}$

10). $f(x) = \sqrt{x+8} + 3$; at $x = 1$

$\lim_{x \to 1} \frac{(\sqrt{x+8} + 3) - (\sqrt{1+8} + 3)}{x - 1}$

$\lim_{x \to 1} \frac{\sqrt{x+8} + 3 - 6}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x+8} - 3}{x - 1}$

$\lim_{x \to 1} \frac{\sqrt{x+8} - 3}{x - 1} \cdot \frac{(\sqrt{x+8} + 3)}{(\sqrt{x+8} + 3)}$

$\lim_{x \to 1} \frac{x + 8 - 9}{(x-1)(\sqrt{x+8} + 3)}$

$\lim_{x \to 1} \frac{x - 1}{(x-1)(\sqrt{x+8} + 3)}$

$\lim_{x \to 1} \frac{1}{\sqrt{x+8} + 3} = \frac{1}{\sqrt{1+8} + 3}$

$\boxed{\frac{1}{6}}$

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★ Find find the instan. Rat of change at the given x. Use that slope in the point-slope form.

III. Find the tangent line of each at the given x value.

11). $f(x) = 2x - 1$; at $x = -35$

$$\lim_{x \to -35} \frac{(2x-1)-(2(-35)-1)}{x-(-35)}$$

$$\frac{2x-1-(-71)}{x+35}$$

$$\lim_{x \to -35} \frac{2x+70}{x+35}$$

$$\lim_{x \to -35} \frac{2\cancel{(x+35)}}{\cancel{(x+35)}}$$

$$\lim_{x \to -35} 2 = \boxed{2} \text{ slope}$$

pt $(-35, f(-35))$ → $\boxed{(-35, -71)}$

$$\boxed{y + 71 = 2(x+35)}$$ EQ

12). $f(x) = x^3$; at $x = -1$

$f(-1) = (-1)^3 = -1$

pt → $\boxed{(-1,-1)}$

$$\lim_{x \to -1} \frac{x^3 - f(-1)}{x-(-1)}$$

$$\lim_{x \to -1} \frac{x^3-(-1)}{x+1}$$

$$\lim_{x \to -1} \frac{x^3+1}{x+1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{(x+1)}}$$

$$\lim_{x \to -1} x^2-x+1$$

$= (-1)^2 - (-1) + 1$

$= 3$ ← slope

$$\boxed{y+1 = 3(x+1)}$$ EQ

13). $f(x) = x^3 + x^2 - 2x - 3$; at $x = -2$

$f(-2) = (-2)^3 + (-2)^2 - 2(-2) - 3$

$f(-2) = -8 + 4 + 4 - 3 = -3$

pt $\boxed{(-2,-3)}$

$$\lim_{x \to -2} \frac{x^3+x^2-2x-3-(-3)}{x-(-2)}$$

$$\lim_{x \to -2} \frac{x^3+x^2-2x}{x+2}$$

$$\lim_{x \to -2} \frac{x(x^2+x-2)}{(x+2)}$$

$$\lim_{x \to -2} \frac{x\cancel{(x+2)}(x-1)}{\cancel{(x+2)}}$$

$$\lim_{x \to -2} x(x-1) = -2(-2-1) = 6 \text{ slope}$$

EQ $\boxed{y+3 = 6(x+2)}$

14). $f(x) = \sqrt{x-3} + 6$; at $x = 12$

$f(12) = \sqrt{12-3} + 6 = 9$ pt $\boxed{(12, 9)}$

$$\lim_{x \to 12} \frac{\sqrt{x-3}+6-9}{x-12} = \lim_{x \to 12} \frac{\sqrt{x-3}-3}{x-12}$$

$$\lim_{x \to 12} \frac{\sqrt{x-3}-3}{x-12} \cdot \frac{\sqrt{x-3}+3}{\sqrt{x-3}+3}$$

$$\lim_{x \to 12} \frac{x-3-9}{(x-12)(\sqrt{x-3}+3)}$$

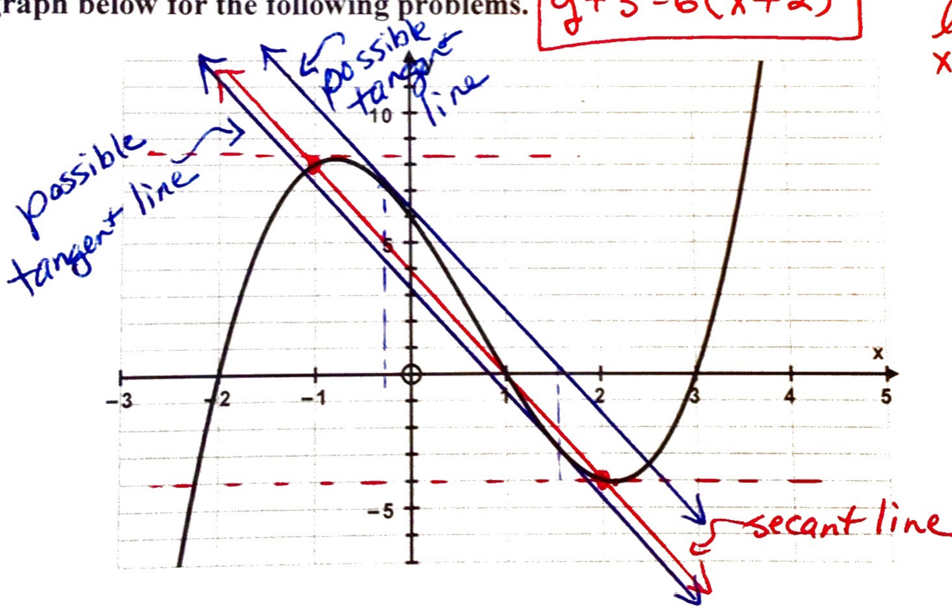
$$\lim_{x \to 12} \frac{\cancel{x-12}}{\cancel{(x-12)}(\sqrt{x-3}+3)}$$

$$\lim_{x \to 12} \frac{1}{\sqrt{x-3}+3} \text{ slope}$$

$$= \frac{1}{\sqrt{12-3}+3} = \boxed{\frac{1}{6}}$$

$$\boxed{y - 9 = \tfrac{1}{6}(x-12)}$$

IV. Use the graph below for the following problems.



15). Use a rule to draw the secant line from $x = -1$ to $x = 2$ on/for the above function.

16). Estimate at least one value of x in the interval $[-1, 2]$ that would have a tangent line that is parallel to the secant line drawn in #15. Graph the tangent line. $x \approx 0.25$ or $x \approx 1.5$

17). Estimate what values of x (if any) would have tangent lines with slopes of zero.

$x = -.8$, $x \approx 2.1$

18). Estimate what value of x would have the largest (positive or negative) slope in the interval $[-1, 2]$.

$x \approx 1$