WS_Derivative Practice Calculus AB Name Key

Find the derivative of each function. **Simplify the answers.**

1. $f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - 6x^2 + 8$

$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 - 6x^2 + 8$

$\boxed{f'(x) = x^3 + 2x^2 - 12x}$

2. $y = \frac{3}{x^5} - \frac{7}{x^3} + 6\sqrt{x}$

$y = 3x^{-5} - 7x^{-3} + 6x^{\frac{1}{2}}$

$y' = -15x^{-6} + 21x^{-4} + 3x^{-\frac{1}{2}}$

$\boxed{y' = -\frac{15}{x^6} + \frac{21}{x^4} - \frac{3}{\sqrt{x}}}$

3. $f(x) = (x^3 + 2)(4x^2 - x + 1)$

$f'(x) = (x^3+2)(8x-1) + (4x^2-x+1)(3x^2)$

$f'(x) = 8x^4 - x^3 + 16x - 2 + 12x^4 - 3x^3 + 3x^2$

$f'(x) = \boxed{20x^4 - 4x^3 + 3x^2 + 16x - 2}$

4. $h(x) = \frac{3x^4-5}{x^3+4}$

$h'(x) = \frac{(x^3+4)(12x^3) - (3x^4-5)(3x^2)}{(x^3+4)^2}$

$h'(x) = \frac{12x^6 + 48x^3 - 9x^6 + 15x^2}{(x^3+4)^2}$

$h'(x) = \boxed{\frac{3x^6 + 48x^3 + 15x^2}{(x^3+4)^2}}$

5. $f(x) = 3x + x\tan x$

$\boxed{f'(x) = 3 + x\sec^2 x + \tan x}$

6. $g(x) = \frac{\cos x}{1+\sin x}$

$g'(x) = \frac{(1+\sin x)(-\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$

$g'(x) = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$

$g'(x) = \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1+\sin x)^2}$

$g'(x) = \frac{-\sin x - 1}{(1+\sin x)^2}$

$g'(x) = \frac{-(\sin x + 1)}{(1+\sin x)^2} = \boxed{-\frac{1}{1+\sin x}}$

7. Find $\frac{d^2P}{dt^2}$, if $P = t^5 - 2t^3 + 4t^2 - t$

$\frac{dP}{dt} = 5t^4 - 6t^2 + 8t - 1$

$\boxed{\frac{d^2P}{dt^2} = 20t^3 - 12t + 8}$

8. Find $y'''(-2)$ given $y = 2t^5 - t^3 + 4t^2 - 7t$ note change

$y' = 10t^4 - 3t^2 + 8t - 7$

$y'' = 40t^3 - 6t + 8$

$y''' = 120t^2 - 6$

$y'''(-2) = 120(-2)^2 - 6$

$480 - 6 = \boxed{474}$

9. Write the equation for the lines that are tangent and normal to the graph of $y = x^2 \sin x$ at $x = \frac{\pi}{2}$.

$y' = x^2 \cos x + (\sin x)2x$

$y'(\frac{\pi}{2}) = (\frac{\pi}{2})^2 \cos\frac{\pi}{2} + 2(\frac{\pi}{2})\sin\frac{\pi}{2}$

$y'(\frac{\pi}{2}) = \frac{\pi^2}{4}(0) + \pi(1)$

$y'(\frac{\pi}{2}) = \pi$

$y(\frac{\pi}{2}) = (\frac{\pi}{2})^2 \sin\frac{\pi}{2}$

$y(\frac{\pi}{2}) = \frac{\pi^2}{4}(1)$

$(\frac{\pi}{2}, \frac{\pi^2}{4})$

Tangent Line

$\boxed{y - \frac{\pi^2}{4} = \pi(x - \frac{\pi}{2})}$

Normal Line

$\boxed{y - \frac{\pi^2}{4} = -\frac{1}{\pi}(x - \frac{\pi}{2})}$

10. Suppose f and g are differentiable with values as shown.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	4	5	6

Find the derivative at x =2 for the following functions. Show all work.

a. $h(x) = f(x) + g(x)$ $h'(x) = f'(x) + g'(x)$

$h'(2) = f'(2) + g'(2)$

$h'(2) = 5 + 6$

$\boxed{h'(2) = 11}$

b. $j(x) = f(x) \cdot g(x)$ $j'(x) = f(x) \cdot g'(x) + g(x) f'(x)$

$j'(2) = f(2) \cdot g'(2) + g(2) \cdot f'(2)$

$j'(2) = 3 \cdot 6 + 4 \cdot 5$

$j'(2) = 18 + 20 = \boxed{38}$

c. $m(x) = \dfrac{f(x)}{g(x)}$ $m'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$m'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$

$m'(2) = \dfrac{4(5) - 3(6)}{4^2} = \dfrac{20-18}{16}$

$\dfrac{2}{16} = \boxed{\dfrac{1}{8}}$

d. $n(x) = x \cdot f(x)$

$m'(x) = x f'(x) + [f(x)]1$

$m'(2) = 2f'(2) + f(2)$

$2(5) + 3$

$10 + 3 = \boxed{13}$

e. $p(x) = \dfrac{x^2}{g(x)}$

$p'(x) = \dfrac{[g(x)](2x) - x^2 g'(x)}{[g(x)]^2}$

$p'(2) = \dfrac{[g(2)](2 \cdot 2) - 2^2 g'(2)}{[g(2)]^2} = \dfrac{4 \cdot 4 - 4(6)}{4^2} = \dfrac{16-24}{16} = \dfrac{-8}{16}$

$\boxed{-\frac{1}{2}}$

11. $f(x) = \sqrt{x} g(x)$ $g(4) = 2$ $g'(4) = 3$

$f'(4) =$

$f'(x) = \sqrt{x}\, g'(x) + [g(x)](\frac{1}{2}x^{-\frac{1}{2}})$

$f'(4) = \sqrt{4}\, g'(4) + [g(4)](\frac{1}{2\sqrt{4}})$

$2(3) + 2(\frac{1}{4})$

$6 + \frac{1}{2} = \boxed{6\frac{1}{2} \text{ or } \frac{13}{2}}$