WS: Limits

Worksheet #1

Ans. Key

For questions 1 and 2, find the value of a so that the limit exists:

1.) as x approaches 2 for $f(x)=\begin{cases} a-x^2, & x<2 \\ x^2+5x-3, & x\geq 2 \end{cases}$

$\lim_{x\to 2^-} a-x^2 = \lim_{x\to 2^+} x^2+5x-3$

$a-(2)^2 = (2)^2+5(2)-3$

$a-4 = 11$

$\boxed{a=15}$

2.) as x approaches –1 for $f(x)=\begin{cases} x^3-4x, & x<-1 \\ 2x+a, & x\geq -1 \end{cases}$

$\lim_{x\to -1^-} x^3-4x = \lim_{x\to -1^+} 2x+a$

$(-1)^3-4(-1) = 2(-1)+a$

$-1+4 = -2+a$

$3 = -2+a$

$\boxed{5=a}$

3.) Use the Sandwich Theorem to show that

$\lim_{x\to 0} \dfrac{x\sin x}{2-2\cos x} = 1$ if $1-\dfrac{x^2}{6} < \dfrac{x\sin x}{2-2\cos x} < 1$

$\lim_{x\to 0} 1-\frac{x^2}{6}$ $\qquad$ $\lim_{x\to 0} 1$

$1-\frac{0^2}{6}$ $\qquad$ 1

$1 \qquad = \qquad 1$

$\therefore \lim_{x\to 0} \frac{x\sin x}{2-2\cos x}$ must also $= 1$