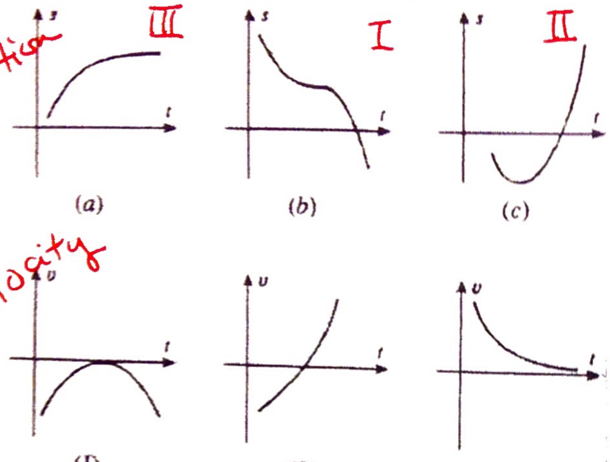
WE_Unit 2_Review Calculus AB Name Key

1. For the graphs in the accompanying figure, match the position functions (a) to (c) with their corresponding velocity functions (I) to (III).

position

III I II

velocity



(a) (b) (c)

(I) (II) (III)

2.

Use the graph of $y = f(x)$ in the accompanying figure to estimate the value of $f'(1)$, $f'(3)$, $f'(5)$, and $f'(6)$.

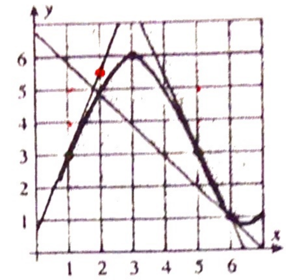


Figure Ex-1

$f'(1) = 2\frac{1}{2} = \frac{5}{2}$

$f'(3) = 0$

$f'(5) = -2\frac{1}{2} = -\frac{5}{2}$

$f'(6) = -1$

3.

For the function graphed in the accompanying figure, arrange the numbers 0, $f'(-3)$, $f'(0)$, $f'(2)$, and $f'(4)$ in increasing order.

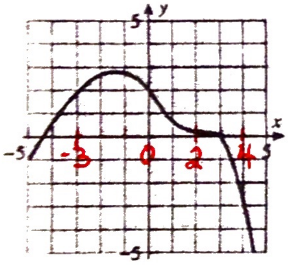


Figure Ex-2

$f'(4)$ Smallest

$f'(0)$

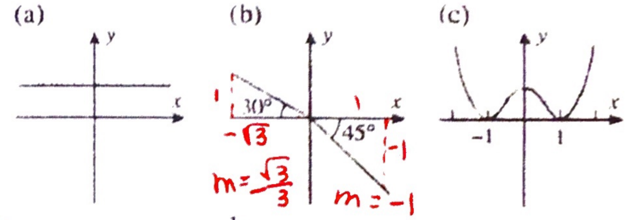
$f'(2)$

0

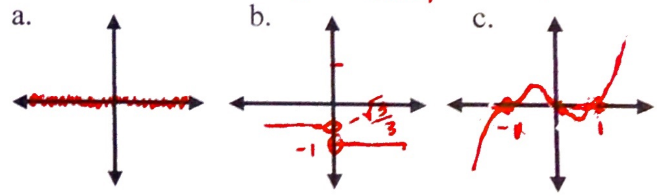
$f'(-3)$ largest

4. Sketch the graph of the derivative function for each function below.

(a) (b) (c)



a. b. c.



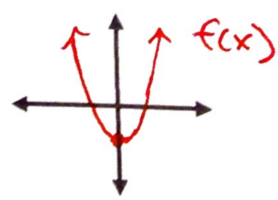
5.

Given that the tangent line to the graph of $y = f(x)$ at the point (2,5) has the equation $y = 3x - 1$, find $f'(2)$. $f'(2) = 3$

For this function, what is the instantaneous rate of change of y with respect to x at $x = 2$?

3

6. Sketch the graph of a functions f for which $f(0) = -1$, $f'(0) = 0$, $f'(x) < 0$ if $x < 0$ and $f'(x) > 0$ if $x > 0$



7. Given that $f(3) = -1$ and $f'(3) = 5$. Find an equation for the tangent line to the graph of $y = f(x)$ at $x = 3$.

$m = f'(3) = 5$

pt $(3, -1)$

$(3, f(-1))$

$y + 1 = 5(x - 3)$

or

$y = 5(x-3) - 1$

or

$y = 5x - 16$

8. Given that the tangent line to $y = f(x)$ at the point (1, 2) passes through the point (3, 5). Find $f'(1)$.

$f'(1) = \frac{5-2}{3-1} = \boxed{\frac{3}{2}}$

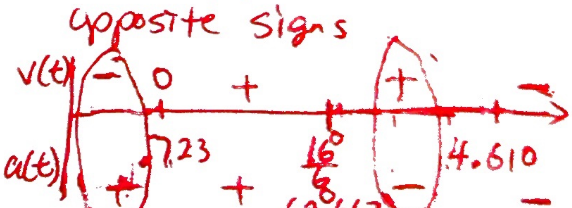
Key

General Derivative	Derivative at a Point $x = a$
$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$	$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ or $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

9. If $f'(a) = \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$, what is $f(x)$? and what is a ? $f(x) = \sqrt{x}$, $a = 9$	10. If $f'(a) = \lim_{h\to 0} \frac{(-1+h)^2-1}{h}$, what is $f(x)$? and what is a ? $f(x) = x^2$, $a = -1$
11. *Calculate* $\frac{dy}{dt}$ *, if* $y = \sqrt{t}\cot t$ Product Rule $y = t^{\frac{1}{2}} \cot t$ $\frac{dy}{dt} = t^{\frac{1}{2}} \frac{d}{dt}(\cot t) + \cot t \frac{d}{dt}(t^{\frac{1}{2}})$ $t^{\frac{1}{2}}(-\csc^2 t) + \cot t (\frac{1}{2} t^{-\frac{1}{2}})$ $\boxed{-\sqrt{t}\csc^2 t + \frac{1}{2\sqrt{t}}\cot t}$	12. Find the equation of the tangent line to the curve $y = 2 + 3\cos x$ *at* $(\pi, -1)$ $y' = -3\sin x$ $y'(\pi) = -3\sin\pi$ $y'(\pi) = -3(0)$ $y'(\pi) = 0$ $y + 1 = 0(x - \pi)$ $\boxed{y = -1}$

Calculator is Permitted for the problem below.

13. A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by the function $s(t) = -t^3 + 8t^2 - 10t + 7$ where s is measured in meters and t is measured in seconds.

a. Find the particle's instantaneous velocity at any time t. $v(t) = s'(t) = -3t^2 + 16t - 10$	b. Find the particle's acceleration at any time t. $a(t) = v'(t) = s''(t) = -6t + 16$
c. When is the particle at rest? Justify your answer. when $v(t) = 0$ Find zeros on graph at $t = .723$ sec or $t = 4.610$ sec	d. Find the displacement of the particle from $t = 0\,sec$ *to* $t = 5\,sec$. Show set up. $s(5) - s(0)$ $32 - 7 = \boxed{25\ m}$
e. Find the total distance the particle traveled from $t = 0\ sec\ to\ t = 5\ sec$. Show set up. $\lvert s(.723) - s(0)\rvert + \lvert s(4.610) - s(.723)\rvert$ $+ \lvert s(5) - s(4.610)\rvert$ $= 33.889\ m$	f. What is the particle's speed at 7 seconds? $\lvert v(7)\rvert = \lvert -45\rvert$ $= 45\ m/sec$ g. When is the particle's speed decreasing? Justify your answer. when $v(x)$ and $a(x)$ have opposite signs  $(0, .723)$ sec or $(2.667, 4.610)$ sec

where $a(t) = 0$
$0 = -6t + 16$
$t = \frac{16}{6}$
≈ 2.667