Review WS - Unit 3

1) $y = \sin^{-1}\sqrt{2x}$ $\qquad \frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{2x})^2}}\frac{d}{dx}(2x)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-2x}} \cdot \frac{1}{2}(2x)^{-\frac{1}{2}}(2)$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{2x}\sqrt{1-2x}}} = \frac{1}{\sqrt{2x(1-2x)}}$$

2) $y = \log_7(x^2+6)$ $\qquad \frac{d}{dx}\log_a u = \frac{1}{\ln a}\cdot\frac{1}{u}\cdot\frac{du}{dx}$

$$\frac{dy}{dx} = \frac{1}{\ln 7}\cdot\frac{1}{x^2+6}\cdot\frac{d(x^2+6)}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{(\ln 7)(x^2+6)}}$$

3) $y = \sin^2(3x)$ $\qquad \frac{d}{dx}\sin u = \cos u\frac{du}{dx}$

$$\frac{dy}{dx} = 2\sin(3x)\frac{d}{dx}(\sin 3x)$$

$$\frac{dy}{dx} = 2\sin(3x)[\cos(3x)](3)$$

$$\boxed{\frac{dy}{dx} = 6\sin(3x)\cos 3x}$$

4) $y = e^{\tan x}$ $\qquad \frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$

$$\frac{dy}{dx} = e^{\tan x}\frac{d}{dx}\tan x$$

$$\boxed{\frac{dy}{dx} = e^{\tan x}(\sec^2 x)}$$

5) $y = \cot^{-1}(x^3)$ $\qquad \frac{d}{dx}\cot^{-1}u = -\frac{1}{1+u^2}\cdot\frac{du}{dx}$

$$\frac{dy}{dx} = -\frac{1}{1+(x^3)^2}\frac{d(x^3)}{dx}$$

$$\boxed{\frac{dy}{dx} = -\frac{3x^2}{1+x^6}}$$

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6) $y = \ln(e^{4x-7})$ $\qquad \frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$

$\frac{dy}{dx} = \frac{1}{e^{4x-7}} \cdot \frac{d(e^{4x-7})}{dx}$ $\qquad \frac{d}{dx}e^{u} = e^{u}\frac{du}{dx}$

$= \frac{1}{e^{4x-7}}(e^{4x-7})\frac{d}{dx}(4x-7)$

$= \boxed{4}$

7) $y = 4^{\sqrt{x}}$ $\qquad \frac{d}{dx}a^{u} = a^{u}\ln a\,\frac{du}{dx}$

$\frac{dy}{dx} = 4^{\sqrt{x}}(\ln 4)\frac{d}{dx}(x)^{\frac{1}{2}}$

$\frac{dy}{dx} = 4^{\sqrt{x}}(\ln 4)(\frac{1}{2}x^{-\frac{1}{2}})$

$\boxed{\frac{dy}{dx} = \frac{4^{\sqrt{x}}(\ln 4)}{2\sqrt{x}}}$

8) $y = (3x^4-7)^3$

$\frac{dy}{dx} = 3(3x^4-7)^2\frac{d}{dx}(3x^4-7)$

$\frac{dy}{dx} = 3(3x^4-7)^2(12x^3)$

$\boxed{\frac{dy}{dx} = 36x^3(3x^4-7)^2}$

9) $f(x) = g(r(x))$, $g(x) = \frac{3}{x^4}$, $r(x) = \cos x$

$f'(x) = g'(r(x))\cdot r'(x)$ $\qquad g(x) = 3x^{-4}$

$= g'(\cos x)(-\sin x)$ $\qquad g'(x) = -12x^{-5}$

$= -12(\cos x)^{-5}(-\sin x)$

$= \boxed{\frac{12\sin x}{\cos^5 x}}$

or $f(x) = g(\cos x) = \frac{3}{\cos^4 x} = 3\cos^{-4}x$

$f'(x) = -12\cos^{-5}x\,\frac{d}{dx}\cos x$

$= -12\cos^{-5}x(-\sin x)$

$= \frac{-12\sin x}{\cos^5 x}$

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10) $\frac{dy}{dx} = ?$ $x^3y - xy^3 + 3y = 4$

$$x^3y' + y(3x^2) - [x(3y^2y') + y^3(1)] + 3y' = 0$$
$$x^3y' + 3x^2y - 3xy^2y' - y^3 + 3y' = 0$$
$$x^3y' - 3xy^2y' + 3y' = y^3 - 3x^2y$$
$$y'(x^3 - 3xy^2 + 3) = y^3 - 3x^2y$$
$$\boxed{y' = \frac{y^3 - 3x^2y}{x^3 - 3xy^2 + 3}}$$

11) $\frac{d^2y}{dx^2} = ?$ $x^3 + y^3 = 9$

$$3x^2 + 3y^2y' = 0$$
$$y' = \frac{-3x^2}{3y^2} = -\frac{x^2}{y^2}$$
$$\frac{d^2y}{dx^2} = \frac{y^2(-2x) - (-x^2)(2yy')}{(y^2)^2}$$
$$\frac{\left[-2xy^2 + 2x^2y\left(-\frac{x^2}{y^2}\right)\right]y}{(y^4)\,y}$$
$$\boxed{\frac{-2xy^3 - 2x^4}{y^5}}$$

12) Eq of line normal to $y^2 + x = 4$ at $(-5, 3)$

$$2yy' + 1 = 0$$
$$y' = -\frac{1}{2y} \qquad \left.\frac{dy}{dx}\right|_{(-5,3)} = -\frac{1}{2(3)} = -\frac{1}{6}$$

$\perp m = 6$, pt $(-5, 3)$ $\boxed{y - 3 = 6(x + 5)}$

13) Use implicit differentiation, find derivative of the inverse of $xy = \sin(x^2) + x$.

$x \leftrightarrow y$ $yx = \sin(y^2) + y$

$$y(1) + xy' = [\cos(y^2)](2yy') + y'$$
$$y = \cos(y^2)(2yy') + y' - xy'$$
$$y = [\cos(y^2)(2y) + 1 - x]y'$$
$$\boxed{y' = \frac{y}{2y\cos y^2 - x + 1}}$$

14) Use inverse rule find derivative of the inverse of $f(x) = \frac{x+1}{2x-3}$ at $x = -2$

$f'(x) = \frac{(2x-3)(1) - (x+1)(2)}{(2x-3)^2} = \frac{2x - 3 - 2x - 2}{(2x-3)^2} = \frac{-5}{(2x-3)^2}$

$f^{-1}(x)' = -\frac{(2y-3)^2}{5}$

$f^{-1}(-2) = -\frac{(2(1)-3)^2}{5}$

$f^{-1}(-2) = -\frac{1}{5} = \boxed{-\frac{1}{5}}$

$f(x) \rightarrow (?, -2)$

$f^{-1}(x) \rightarrow (-2, ?)$

$f^{-1}(x)\ (-2, -1)$

$-2 = \frac{x+1}{2x-3}$

$-4x + 6 = x + 1$

$-5x = -5$

$x = 1$

15) If $h(x) = g^{-1}(x)$, find $h'(2)$ given $h(2) = 6$, $g(2) = 4$

$h\ (2, 6)$ $g\ (6, 2)$ $g'(2) = -3$, $g'(6) = -5$

$f'(a) = \frac{1}{g'(b)}$ $h'(2) = \frac{1}{g'(6)} = \boxed{-\frac{1}{5}}$

16) State which grows faster. Show or explain why

a) $y = \log_3 x$ or $y = \ln x$

same, all log functions grows the same rate.

b) $y = x^{10}$ or $y = 100x^{10}$

$\lim_{x \to \infty} \frac{x^{10}}{100x^{10}} = \frac{1}{100}$

limit is a constant → same

c) $y = x^{10}$ or $y = 10^x$

$y = 10^x$ is faster

exponential grows faster then polynomial function.

d) $y = e^x$ or $y = e^{x+3}$

$\lim_{x \to \infty} \frac{e^x}{e^{x+3}}$

$\lim_{x \to \infty} \frac{e^x}{e^x \cdot e^3} = \frac{1}{e^3}$ same

e) $y = e^x$ or $y = e^{3x}$

$\lim_{x \to \infty} \frac{e^x}{e^{3x}} = \lim_{x \to \infty} \frac{e^x}{e^x \cdot e^{2x}}$

$\lim_{x \to \infty} \frac{1}{e^{2x}} = 0$ $y = e^{3x}$ is faster

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17) use table to find the following

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-2	-4	3	-1	-3
3	1	2	5	7

a) $f-g$ at $x=-2$

$f'(-2)-g'(-2)$

$-1-(-3)=\boxed{2}$

b) fg at $x=-2$

$f(-2)g'(-2)+g(-2)f'(-2)$

$-4(-3)+3(-1)$

$12-3=\boxed{9}$

c) $\frac{f}{g}$ at $x=3$

$$\frac{g(3)f'(3)-f(3)g'(3)}{[g(3)]^2}$$

$$\frac{2(5)-1(7)}{2^2}=\frac{10-7}{4}=\boxed{\frac{3}{4}}$$

d) $f(g(x))$ at $x=-2$

$f'(g(-2))\cdot g'(-2)$

$f'(3)\cdot(-3)$

$5\cdot -3=\boxed{-15}$

e) f^{-1} at $x=1$ so $(1, ?)$

$\frac{1}{f'(y)}=\frac{1}{f'(3)}$ and $(?\ 1)$ is on f

$?=3$

$\boxed{\frac{1}{5}}$