Unit 2A Review
Polynomial Functions

Analysis

Name Key

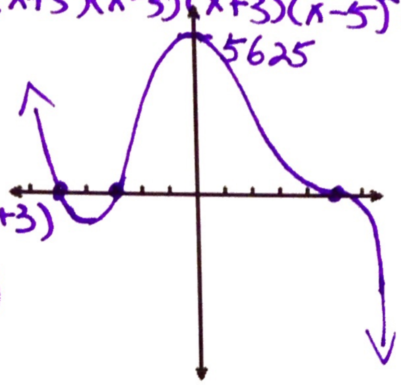
Sketch each polynomial function **WITHOUT** the calculator. Label ALL Intercepts.

1. $f(x) = -3(x^2 - 25)(3 + x)(5 - x)^2$

$n = 5$ ⊖ $\quad -3(x+5)(x-5)(x+3)(x-5)^2 = f(x)$

↖ ↘

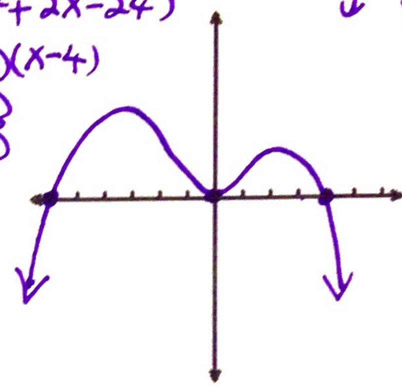
$f(x) =$
$-3(x+5)(x-5)^3(x+3)$
$\{-5, 5, 5, 5, -3\}$



2. $f(x) = -x^4 - 2x^3 + 24x^2$

$n = 4$ ⊖ ↓ ↓

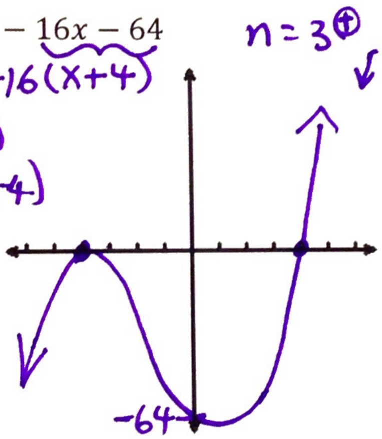
$f(x) = -x^2(x^2 + 2x - 24)$
$f(x) = -x^2(x+6)(x-4)$
$\{0, 0, -6, 4\}$



3. $f(x) = x^3 + 4x^2 - 16x - 64$

$n = 3$ ⊕ ↙ ↗

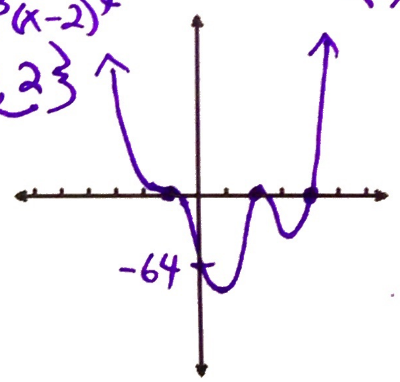
$f(x) = x^2(x+4) - 16(x+4)$
$f(x) = (x+4)(x^2 - 16)$
$f(x) = (x+4)(x+4)(x-4)$
$f(x) = (x+4)^2(x-4)$
$\{-4, -4, 4\}$



4. $f(x) = -(4 - x)(x + 1)^3(x - 2)^4$

$n = 8^+$ ↖ ↗

$f(x) = (x-4)(x+1)^3(x-2)^4$
$\{4, -1, -1, -1, 2, 2, 2, 2\}$



5. Given one root of the polynomial function:
   a. Find the remaining roots of the function. Must **SHOW ALL WORK!!**
   b. Sketch the graph of the function. Label ALL intercepts. **NO CALCULATOR**.
   c. Write $f(x)$ in factored form.

$f(x) = x^3 - 7x^2 - 5x + 75\ ; \ x = 5$

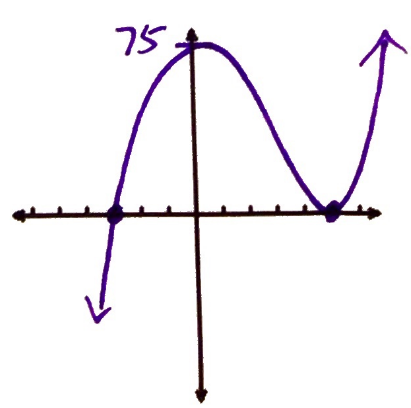
a. Roots: $\{5, 5, -3\}$

b. Sketch

c. Factors: $f(x) = (x-5)^2(x+3)$

5	1	-7	-5	75
		5	-10	-75
	1	-2	-15	0

$f(x) = (x-5)(x^2 - 2x - 15)$
$f(x) = (x-5)(x-5)(x+3)$



6. *Given:* $f(x) = x^3 - 4x^2 - 5x + 14$

a. List all possible zeros for $f(x)$.

$\pm 1, \pm 2, \pm 7, \pm 14$

b. Use Descartes' Rule of signs to determine the number of possible positive, negative and imaginary zeros.

$f(x) = + \; - \; - \; + \rightarrow 2^{\oplus}$ vars $= 2$ or 0

$f(-x) = - \; - \; + \; + \rightarrow 1^{\ominus}$ vars $= 1$

+	−	i
2	1	0
0	1	2

c. Find ALL roots. Must show work.

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -5 & 14 \\ & & -2 & 12 & -14 \\ \hline & 1 & -6 & 7 & \| \; 0 \end{array}$$

$f(x) = (x+2)(x^2 - 6x + 7)$

$\{-2, \; 3 \pm \sqrt{2}\}$

$a = 1$

$b = -6$

$c = 7$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 - 28}}{2}$

$x = \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2}$

$= \frac{\cancel{2}(3 \pm \sqrt{2})}{\cancel{2}}$

#7 – 8. Use synthetic Division to find the **quotient** and the **remainder** for each problem.

7. $(5x^3 + 18x^2 - 2x - 8) \div (x + 4)$

$$\begin{array}{r|rrrr} -4 & 5 & 18 & -2 & -8 \\ & & -20 & 8 & -24 \\ \hline & 5 & -2 & 6 & \| -32 \end{array}$$

$Q(x) = 5x^2 - 2x + 6 \quad R(x) = -32$

8. $\frac{x^4 + 6x^2 - 10}{x - 2}$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 6 & 0 & -10 \\ & & 2 & 4 & 20 & 40 \\ \hline & 1 & 2 & 10 & 20 & \| \; 30 \end{array}$$

$Q(x) = x^3 + 2x^2 + 10 + 20 \quad R(x) = 30$

Evaluate using the REMAINDER THEOREM. Show work.

9. $f(x) = x^5 - 3x^4 + 8x^2 - 9x + 27$ $\boxed{f(-2) = -3}$
Find $f(-2)$.

$$\begin{array}{r|rrrrrr} -2 & 1 & -3 & 0 & 8 & -9 & 27 \\ & & -2 & 10 & -20 & 24 & -30 \\ \hline & 1 & -5 & 10 & -12 & 15 & \| -3 \end{array}$$

10. $f(x) = -3x^4 + 10x^3 + 8x - 5$ $\boxed{f(4) = -101}$
Evaluate $f(4)$.

$$\begin{array}{r|rrrrr} 4 & -3 & 10 & 0 & 8 & -5 \\ & & -12 & -8 & -32 & -96 \\ \hline & -3 & -2 & -8 & -24 & \| -101 \end{array}$$

11. Use the Factor Theorem to determine whether $(x - 3)$ is a factor of $2x^3 - x^2 + 3$.
EXPLAIN Why or Why Not?

$$\begin{array}{r|rrrr} 3 & 2 & -1 & 0 & 3 \\ & & 6 & 15 & 45 \\ \hline & 2 & 5 & 15 & \| \; 48 \end{array}$$

No, $x-3$ is not a factor b.c. $f(3) = 48 \neq 0$.

12. If the roots of $f(x)$ *are* $\left\{-8, \frac{4}{9}, 5\right\}$, express $f(x)$ as factors. (NO Fractions in factors)

$f(x) = (x+8)(9x-4)(x-5)$

13. Given a polynomial function of degree 5 and roots of : $6, -\sqrt{10}, 3+2i$. Find the remaining roots.

Remaining Roots: $\sqrt{10}$, $3-2i$

14. List all possible rational zeros for $f(x) = 5x^4 - 2x^3 + 4x - 8$.

$\pm 1, \pm 2, \pm 4, \pm 8$

$\pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{4}{5}, \pm \frac{8}{5}$

15. Solve $2x^3 + 11x^2 - 7x - 6 = 0$, *given that* $(x+6)$ is a factor of the polynomial.

-6	2	11	-7	-6
		-12	6	6
	2	-1	-1	0

$f(x) = (x+6)(2x^2 - x - 1)$

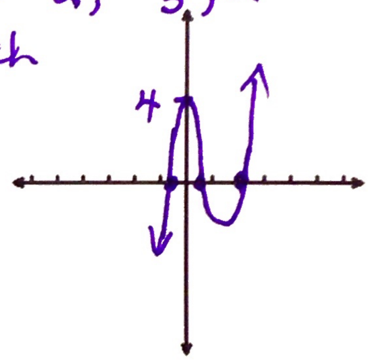
$f(x) = (x+6)(2x+1)(x-1)$

* $\boxed{\text{Solutions } \{-6, -\tfrac{1}{2}, 1\}}$

16. Given: $f(x) = 6x^3 - 11x^2 - 4x + 4$
   a. Factor $f(x)$ completely.
   b. Find the remaining roots.
   c. Sketch the graph.

b) Roots: $x = \frac{1}{2}, -\frac{2}{3}, 2$

c) Sketch



$\frac{1}{2}$	6	-11	-4	4
		3	-4	-4
	6	-8	-8	0

$6x^2 - 8x - 8$

$2(3x^2 - 4x - 4)$

$\boxed{f(x) = 2(2x-1)(3x+2)(x-2)}$ a) Factors

17. Given: $f(x) = x^4 - 3x^3 - 5x^2 + 13x + 6$

* Must do 2 Division to depressed $f(x)$ to a quadratic

a. List all possible rational zeros for $f(x)$.

$\pm 1, \pm 2, \pm 3, \pm 6$

$\boxed{\text{Solutions: } \{3, -2, 1+\sqrt{2}, 1-\sqrt{2}\}}$

b. Solve: $x^4 - 3x^3 - 5x^2 + 13x + 6 = 0$

3	1	-3	-5	13	6
		3	0	-15	-6
-2	1	0	-5	-2	0
		-2	4	2	
	1	-2	-1	0	

$a = 1$
$b = -2$
$c = -1$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2}$

$x = \dfrac{2 \pm \sqrt{4+4}}{2}$

$x = \dfrac{2 \pm \sqrt{8}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2}$

$x = 1 \pm \sqrt{2}$

$(x-3)(x+2)[x-(1+\sqrt{2})][1-(1-\sqrt{2})] = f(x)$