Unit 1_Review Calculus Name Key

Evaluate each limit.

1. $\lim\limits_{x\to3}\frac{x^2-x-6}{x-3} = \boxed{5}$

$f(3) = \frac{3^2-3-6}{3-3} = \frac{0}{0}$

$\lim\limits_{x\to3}\frac{(x-3)(x+2)}{(x-3)}$

$\lim\limits_{x\to3} x+2 = 3+2$

$\boxed{5}$

2. $\lim\limits_{x\to\infty}\frac{5x^3-3x^2}{2-x} = \boxed{-\infty}$

$\frac{5x^3}{-x} = -5x^2$

$\lim\limits_{x\to\infty} 5x^2$

$\boxed{\infty}$

3. $\lim\limits_{x\to5^-}\frac{x+2}{x-5} = \boxed{-\infty}$

$f(5) = \frac{5+2}{5-5} = \frac{7}{0}$

Vertical Asymptote at $x=5$.

Use test point.

Left of $x=5$ $x=4.9$

$f(4.9) = \frac{4.9+2}{4.9-5}$

$\frac{+}{-} = \boxed{-\infty}$

4. $\lim\limits_{x\to\infty}\frac{\sqrt{4x^2+5}}{x^2} = \boxed{0}$

$\frac{\sqrt{4x^2}}{x^2}$

$\frac{2x}{x^2}$

$\lim\limits_{x\to\infty}\frac{2}{x} = 0$

5. $\lim\limits_{x\to2}\frac{x-2}{\frac{1}{x}-\frac{1}{2}} = \boxed{-2}$

$\lim\limits_{x\to2}\frac{x-2}{\frac{1}{x}-\frac{1}{2}}\cdot\frac{2x}{2x}$

$\lim\limits_{x\to2}\frac{2x(x)-2(2x)}{2x(\frac{1}{x})-\frac{1}{2}(2x)}$

$\lim\limits_{x\to2}\frac{2x^2-4x}{2-x}$

$\lim\limits_{x\to2}\frac{-2(-x+\cancel{2})}{(\cancel{2}-x)} = \boxed{-2}$

6. $\lim\limits_{x\to-\infty}\frac{6x+x^2-5x^4}{3x^2-x+8} = \boxed{-\infty}$

$\lim\limits_{x\to-\infty}\frac{-5x^4}{3x^2}$

$\lim\limits_{x\to-\infty} -\frac{5}{3}x^2 = \boxed{-\infty}$

$-\frac{5}{3}(-\infty)^2$

7. $\lim\limits_{h\to0}\frac{\sqrt{4+h}-2}{h} = \boxed{\frac{1}{4}}$

$\lim\limits_{h\to0}\frac{\sqrt{4+h}-2}{h}\cdot\frac{\sqrt{4+h}+3}{\sqrt{4+h}+2}$

$\lim\limits_{h\to0}\frac{4+h-4}{h(\sqrt{4+h}+2)}$

$\lim\limits_{h\to0}\frac{\cancel{h}\,1}{\cancel{h}(\sqrt{4+h}+2)}$ $\quad \frac{1}{2+2} = \boxed{\frac{1}{4}}$

$\lim\limits_{h\to0}\frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4+0}+2}$

8. $\lim\limits_{x\to3}\frac{x-3}{\sqrt{x+1}-2} = \boxed{4}$

$\lim\limits_{x\to3}\frac{x-3}{\sqrt{x+1}-2}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$\lim\limits_{x\to3}\frac{(x-3)(\sqrt{x+1}+2)}{x+1-4}$

$\lim\limits_{x\to3}\frac{(\cancel{x-3})(\sqrt{x+1}+2)}{(\cancel{x-3})}$

$\sqrt{3+1}+2$

$\sqrt{4}+2$

$2+2$

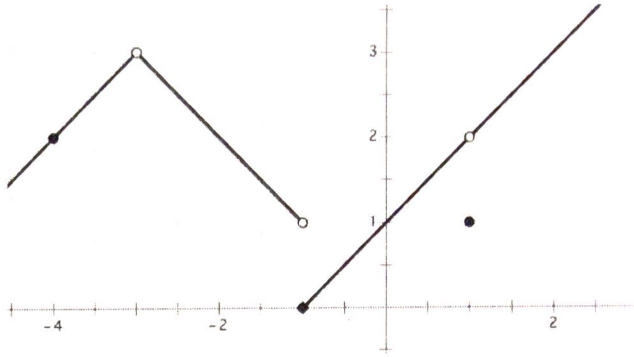
$\boxed{4}$

9. $\lim\limits_{x\to4^-}\frac{|x-4|}{x-4} = \boxed{-1}$

omit this problem

#10 – 17. The graph of $y = f(x)$ is given below.

Evaluate each of the following:



10. $\lim_{x \to -2} f(x) = 2$

11. $\lim_{x \to 2} f(x) = 3$

12. $\lim_{x \to -1^-} f(x) = 1$

14. $\lim_{x \to -1^+} f(x) = 0$

15. $\lim_{x \to -1} f(x) =$ DNE

16. $\lim_{x \to 1} f(x) = 2$

17. Identify all points of discontinuity on the graph above.
State what type of discontinuity at each location and justify why it is discontinuous.

$x = -3$ point (removable) b.c $f(-3)$ does not exists	$x = -1$ Jump b.c. $\lim_{x \to 1} f(x) =$ DNE	$x = 1$ point (removable) b.c. $\lim_{x \to 1} f(x) = 2 \neq 1 = f(1)$

18. State the 3 conditions for continuity at $x = a$.

① $f(a)$ exists

② $\lim_{x \to a} f(x)$ exists

③ $\lim_{x \to a} f(x) = f(a)$

19. Find the value of a as x approaches 2 so that the limit exists.

$$f(x) = \begin{cases} a - x^2, & x < 2 \\ x^2 + 5x - 3, & x \geq 2 \end{cases}$$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\lim_{x \to 2^-} a - x^2 = \lim_{x \to 2^+} x^2 + 5x - 3$

$a - (2)^2 = 2^2 + 5(2) - 3$

$a - 4 = 4 + 10 - 3$

$a = 11 + 4$

$a = \boxed{15}$

20. Find the points of discontinuity of the function $y = \dfrac{x^2 - 2x - 3}{x^2 - 7x + 12}$. For each discontinuity, identify the type of discontinuity (removable, jump, infinite).

$y = \dfrac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x-4)}$

$x \neq 3, x \neq 4$

at $x = 3$ point (removable)

$x = 4$ is a vertical Asymptote

Infinite

21. Find all asymptotes (both vertical and horizontal) for $f(x) = \frac{x-2}{x^2-4}$

$f(x) = \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$f(x) = \frac{1}{x+2}$

Vertical Asymptote

at $\boxed{x = -2}$

Horizontal Asymptote

$\lim_{x\to\infty} \frac{x-2}{x^2-4} = 0$

$\boxed{y=0}$

22. If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$, find $\lim_{x\to c} [f(x)g(x)]$.

(a) $\frac{1}{6}$ (b) $-\frac{1}{3}$ (c) 1

(d) The limit does not exist. (e) None of these

$\lim_{x\to c} [f(x)g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$

$-\frac{1}{2} \cdot \frac{2}{3} = -\frac{1}{6}$

23. Find the limit: $\lim_{x\to 0} \dfrac{\frac{1}{x+3} - \frac{1}{3}}{x}$. $\cdot \frac{3(x+3)}{3(x+3)} = \lim_{x\to 0} \frac{3-1(x+3)}{3x(x+3)} = \lim_{x\to 0} \frac{\cancel{3}-x-\cancel{3}}{3x(x+3)}$

(a) $-\frac{1}{9}$ (b) 0 (c) $\frac{1}{9}$

(d) The limit does not exist. (e) None of these

$\lim_{x\to 0} \frac{-\cancel{x}\ 1}{3\cancel{x}(x+3)}$

$\lim_{x\to 0} -\frac{1}{3(0+3)}$

$-\frac{1}{3\cdot 3} = \boxed{-\frac{1}{9}}$

24. $f(x) = x^2 - 2x + 3$

a. Find the average rate of change for this function on the interval [-1, 3].

Avg Rate $= \frac{f(3)-f(-1)}{3-(-1)} = \frac{6-6}{3+1} = \frac{0}{4} = \boxed{0}$

$f(3) = 3^2 - 2(3) + 3$
$= 9 - 6 + 3$
$f(3) = 6$

$f(-1) = (-1)^2 - 2(-1) + 3$
$= 1 + 2 + 3$
$f(-1) = 6$

b. Find the instantaneous rate of change for this function at $x = 2$.

$\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{(x^2-2x+3)-3}{x-2}$

$f(2) = 2^2 - 2(2) + 3$
$f(2) = 4 - 4 + 3$
$f(2) = 3$

$\lim_{x\to 2} \frac{x^2-2x}{x-2}$

$\lim_{x\to 2} \frac{x\cancel{(x-2)}}{\cancel{(x-2)}} = \boxed{2}$

25. For the function $f(x) = x^2 - 4$ at the point (4, 12), find

(a) the slope of the curve

$$\text{Slope} = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$$

$$\lim_{x \to 4} \frac{x^2 - 4 - 12}{x - 4} = \lim_{x \to 4} \frac{x^2 - 16}{x - 4}$$

$$= \lim_{x \to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}} = \lim_{x \to 4} x + 4 = 4 + 4$$

$$m = \boxed{8}$$

(b) the equation of the tangent line

$m = \text{Slope} = 8$

point $(4, 12)$

$y - y_1 = m(x - x_1)$

$$\boxed{y - 12 = 8(x - 4)}$$

(c) the equation of the normal line.

use slope
opposite reciprocal

$$\boxed{y - 12 = -\frac{1}{8}(x - 4)}$$

(d) Write the equation of the secant line to this curve over the interval $[-3, 1]$.

$$\text{Slope} = \frac{f(1) - f(-3)}{1 - (-3)}$$

$f(1) = (1)^2 - 4$ $\quad$ $f(-3) = (-3)^2 - 4$

$f(1) = -3$ $\quad$ $f(-3) = 5$

pt. $(1, -3)$ $\quad$ pt. $(-3, 5)$

$$\text{slope} = \frac{-3 - 5}{1 + 4} = \frac{-8}{3}$$

$$\boxed{y + 3 = -\frac{8}{3}(x - 1) \quad \text{or} \quad y - 5 = -\frac{8}{3}(x + 3)}$$

26.

x	1.97	1.98	1.99	2	2.01	2.02	2.03
f(x)	3.762	3.787	3.799	3.8	3.801	3.805	3.810

If this is und or even a different value, $\lim_{x \to 2} f(x) = 3.8$

a. Find $\lim_{x \to 2} f(x) = 3.8$

b. Find the average rate of change from $x = 2$ to $x = 2.03$.

$f(2.03) = 3.810$

$f(2) = 3.8$

$$\text{Avg Rate} = \frac{f(2.03) - f(2)}{2.03 - 2}$$

$$\frac{3.810 - 3.8}{.03}$$

$$= \frac{.01}{.03} = \boxed{\frac{1}{3}}$$