Test_Unit 5_Antiderivatives Calculus AB Name: Key

Definite Integrals (Calculator is permitted) Date______

76 possible pts (100) 25%

#1 – 5. Multiple Choice.

1. If $\int_a^b f(x)dx = a + 2b$, then $\int_a^b (f(x) + 5)dx =$

a. $a + 2b + 5$ b. $5b - 5a$ c. $7b - 4a$ d. $7b - 5a$ e. $7b - 6a$

$\int_a^b f(x)\,dx + \int_a^b 5\,dx$ or $\int_a^b 5\,dx = 5x\Big|_a^b = 5b - 5a$

$a + 2b + 5(b-a) = a + 2b + 5b - 5a$

$\boxed{7b - 4a}$

2. $\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$ a. $\frac{x}{\sqrt{1+x^2}}$ b. $\sqrt{1+x^2} - 5$

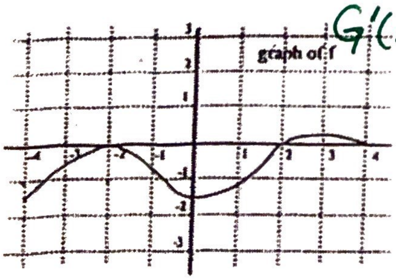
c. $\sqrt{1+x^2}$ d. $\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$ e. $\frac{1}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{5}}$

3. If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x, which of the following is **NOT** necessarily true?

T a. $\int_{-1}^1 f(x)dx > 0$ T b. $\int_{-1}^1 2f(x)dx = 2\int_{-1}^1 f(x)dx$ c. $\int_{-1}^1 f(x)dx = 2\int_0^1 f(x)dx$

T d. $\int_{-1}^1 f(x)dx = -\int_1^{-1} f(x)dx$ T e. $\int_{-1}^1 f(x)dx = \int_{-1}^0 f(x)dx + \int_0^1 f(x)dx$

4. The graph of the function f is shown below:



G'(x)

If the function G is defined by $G(x) = \int_{-4}^x f(t)dt$, for $-4 \le x \le 4$, which of the following statements about G are true?

F **I. G is increasing on (1,2)** $f(x) < 0$ on $(1, 2)$

T **II. G is decreasing on (-4, -3)** $f(x) < 0$ on $(-4, -3)$

T **III. $G(0) < 0$** $G(0) = \int_{-4}^0 f(t)\,dt < 0$

a. none b. II only c. III only d. II and III only e. I, II, and III

8

5. $\frac{d}{dx}\left(\int_0^{x^3} \ln(t^2+1)dt\right) =$

a. $\frac{2x^3}{x^6+1}$

b. $\frac{3x^2}{x^6+1}$

c. $\ln(x^6+1)$

d. $2x^3\ln(x^6+1)$

e. $3x^2\ln(x^6+1)$

$\frac{d}{du}\int_0^u \ln(u^2+1)dt \quad u = x^3, du = 3x^2$

$\ln(u^2+1)du$

$\ln((x^3)^2+1)(3x^2) = 3x^2\ln(x^6+1)$

16

6. Approximate the area under the curve $y = x^2 - 2x + 4$ from $x = 1$ to $x = 5$ by each of the following methods. Set up **4 equal intervals** to find each of the following.
DO NOT EVALUATE!

$\Delta x = \frac{5-1}{4} = 1$

3

a. Left Riemann Sum

$1\left[(1^2-2(1)+4)+(2^2-2(2)+4)+(3^2-2(3)+4)+(4^2-2(4)+4)\right]$

$1\left[f(1)+f(2)+f(3)+f(4)\right] = 1(3+4+7+12) = 27$

2

b. Midpoint Riemann Sum

$1\left[1.5^2-2(1.5)+4)+(2.5^2-2(2.5)+4)+(3.5^2-2(3.5)+4)+(4.5^2-2(4.5)+4)\right]$

$1\left[f(1.5)+f(2.5)+f(3.5)+f(4.5)\right] = \left[3.25+5.25+9.25+15.25\right]$

$\frac{13}{4}+\frac{21}{4}+\frac{37}{4}+\frac{61}{4} = \boxed{33}$

2

c. Trapezoidal Rule

$\frac{1}{2}\left[(1^2-2(1)+4)+2(2^2-2(2)+4)+2(3^2-2(3)+4)+2(4^2-2(4)+4)+(5^2-2(4)+4)\right]$

$\frac{1}{2}\left[f(1)+2f(2)+2f(3)+2f(4)+f(5)\right]$

$\frac{1}{2}\left[3+2(4)+2(7)+2(12)+19\right] = \boxed{34}$

4

7. Approximate the number of gallons of oil spilled given the data in the table below.
Use each of the methods and the number of intervals listed below. **Make sure to show set up.**

Time (hrs)	1	4	6	9	10
Rate (gal/hr)	32	44	36	29	28

a. Right Riemann Sum (4 intervals)

2

$3(44)+2(36)+3(29)+1(28)$

$132+72+87+28 = \boxed{319}$ gal

b. Midpoint Riemann Sum (2 intervals)

2

$\Delta x(44)+\Delta x(29)$

$5(44)+4(29) = 220+116 = \boxed{336}$ gal

12

8. Evaluate the following definite integrals **ALGEBRAICALLY. SHOW ALL WORK!**
Use the calculator **ONLY to check** your answer.
An answer without work will result in ***NO CREDIT*.**

a. $\int_{\frac{\pi}{2}}^{\pi} \cos\left(\frac{1}{2}x\right) dx$ $= \dfrac{\sin\frac{1}{2}x}{\frac{1}{2}}\Big|_{\frac{\pi}{2}}^{\pi}$

1

$2\sin\left(\frac{1}{2}x\right)\Big|_{\frac{\pi}{2}}^{\pi}$

2

$2\sin\left(\frac{1}{2}\pi\right) - 2\sin\left(\frac{1}{2}\cdot\frac{\pi}{2}\right)$

$2\sin\frac{\pi}{2} - 2\sin\frac{\pi}{4}$

1 $2(1) - 2\left(\frac{\sqrt{2}}{2}\right)$ 1

$\boxed{2-\sqrt{2}} \approx .5857864376$

b. $\int_{-2}^{1}(3x+3)dx$ $= \dfrac{3x^2}{2} + 3x \Big|_{-2}^{1}$ 1 1

$\left[\frac{3(1)^2}{2} + 3(1)\right] - \left[\frac{3(-2)^2}{2} + 3(-2)\right]$

$\left[\frac{3}{2} + 3\right] - [6-6]$

$\frac{3}{2} + \frac{6}{2} = \boxed{\frac{9}{2} = 4.5}$

2

19

9. **Solve the differential equation**: $\frac{d^2y}{dx^2} = 20x^3 + 18x$, given that the curve has a slope of 12 at $(1,-2)$.

$\frac{dy}{dx} = 20\left(\frac{x^4}{4}\right) + 18\frac{x^2}{2} + c$

2 $\frac{dy}{dx} = 5x^4 + 9x^2 + c$

$12 = 5(1)^4 + 9(1)^2 + c$

$12 = 5 + 9 + c$

$12 = 14 + c$

$-2 = c$

2

$\frac{dy}{dx} = 5x^4 + 9x^2 - 2$

$y = 5\left(\frac{x^5}{5}\right) + 9\left(\frac{x^3}{3}\right) - 2x + c$

2 $y = x^5 + 3x^3 - 2x + c$

$-2 = 1^5 + 3(1)^3 - 2(1) + c$

$-2 = 1 + 3 - 2 + c$

$-2 = 2 + c$

2 $-4 = c$

1

$\boxed{y = x^5 + 3x^3 - 2x - 4}$

10

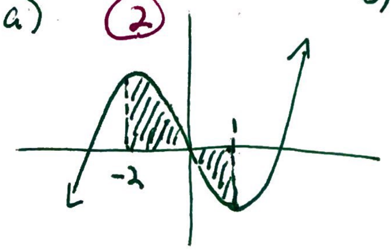
10. Given: $y = x^3 + x^2 - 6x$

a. Sketch and shade the area bounded by the function and the x-axis from $x = -2$ to $x = 1$.

b. Find the exact area of this shaded region using definite integrals with <u>**NO absolute Values**</u>.

Use the calculator to evaluate *ONLY* after you have set up the problem.

a) 2



b) Area $= \int_{-2}^{0}(x^3 + x^2 - 6x)dx - \int_{0}^{1}(x^3 + x^3 - 6x)dx$ 4 4

Did not Negate
-5

-6
No 2 Integrals

$\frac{32}{3} - \left(-\frac{29}{12}\right) = \frac{157}{12}$

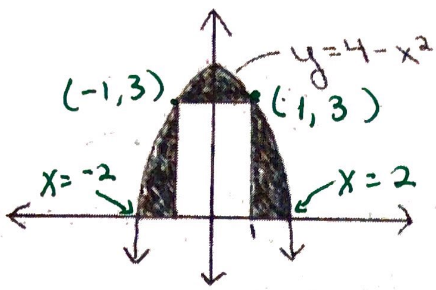
Double Neg
-4

[8.25 on Integ] $10.\bar{6} - (-2.41\bar{6}) \approx 13.08\bar{3}$

28)

9 11. Find the exact area of the shaded region shown below using definite integrals.
Use the calculator to evaluate ONLY after you have set up the problem.

Missing Rectangle Area



$0=4-x^2$

$4=x^2$

$x=\pm 2$ (zeros)

$y=4-1^2$

$y=3$

$y=4-(-1)^3$

$y=3$

$\int_{-2}^{2}(4-x^2)dx - \int_{-1}^{1}3\,dx$

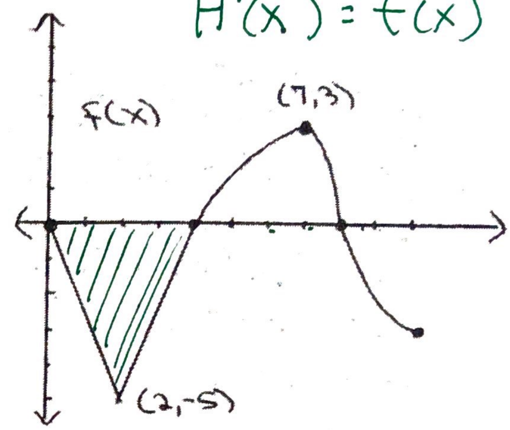
7 $\frac{32}{3}-6=\frac{14}{3}\approx 4.66\bar{7}$ 2

or

$\int_{-2}^{2}(4-x^2)dx-(2\cdot 3)$

12. $H(x)=\int_0^x f(t)dt$ The graph of $f(x)$ is given below.

$H'(x)=f(x)$



3 a. Find $H(0)$ ___0___ $\int_0^0 f(x)dx$

3 b. Find $H(4)$ ___-10___ $\int_0^4 f(x)\,dx$ $\frac{1}{2}(4)(-5)$

3 c. Find $H'(7)$ ___3___ $H'(7)=f(7)$

3 d. Is $H(10)$ positive or negative?

Negative

e. On what interval(s) is $H(x)$ increasing?
2 **Justify your answer.** 1

$H(x)$ is increasing where $H'(x)>0$, since $H'(x)=f(x)$, then $H(x)$ is increasing over $(4,8)$ where $f(x)>0$.

2 f. On what interval(s) is $H(x)$ concave up?

$H(x)$ concave up when $H''(x)>0$; $H''(x)$ is >0 when $H'(x)=f(x)$ is Increasing at $(2,7)$.

2 g. At what value of x does $H(x)$ achieve its maximum?

$H(x)$ achieve its max when $H'(x)$ changes from positive to neg. Since $H'(x)=f(x)$, $H(x)$ has a Max at $\boxed{x=8}$ 1 where $f(x)$ changes from (+) to Neg 1

* Also at $\boxed{x=0}$ endpt.

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