Slope Fields and Differential Equations

Students should be able to:

- Draw a slope field at a specified number of points by hand.
- Sketch a solution that passes through a given point on a slope field.
- Match a slope field to its differential equation.
- Match a slope field to its solution.
- Determine features of the solution to a differential equation based on its slope field and/or its solution.
- Solve separable differential equations.
- Determine a particular solution using an initial condition.
- Model a real world situation using a differential equation.

Multiple Choice

1. (calculator not allowed) If $\frac{dy}{dx} = x^2 y$, then y could be

   (A) $3\ln\left(\frac{x}{3}\right)$

   (B) $e^{\frac{x^3}{3}} + 7$

   (C) $2e^{\frac{x^3}{3}}$

   (D) $3e^{2x}$

   (E) $\frac{x^3}{3} + 1$

2. (calculator not allowed)

   Which of the following is the solutiom to the differential equation $\frac{dy}{dx} = \frac{4x}{y}$,

   where $y(2) = -2$?

   (A) $y = 2x$ for $x > 0$

   (B) $y = 2x - 6$ for $x \neq 3$

   (C) $y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

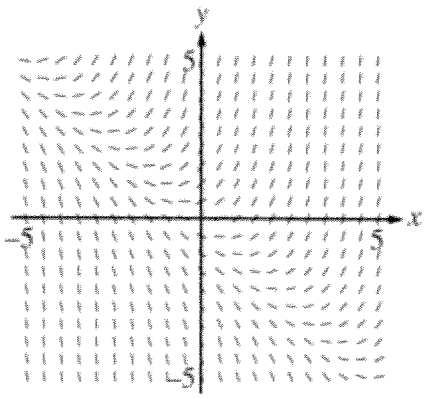
   (D) $y = \sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

   (E) $y = -\sqrt{4x^2 - 6}$ for $x > \sqrt{1.5}$

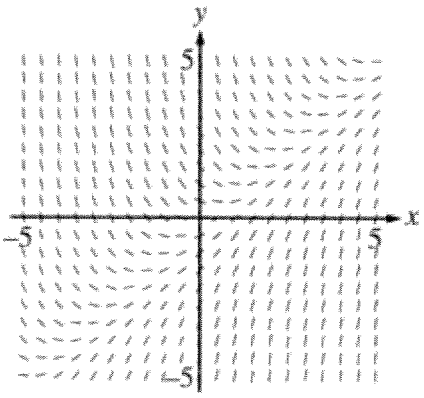
3. (calculator not allowed)

Which of the following is a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

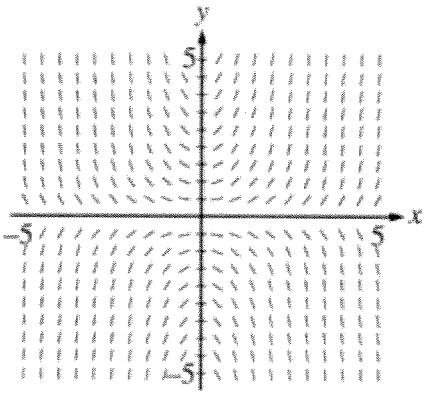
(A)



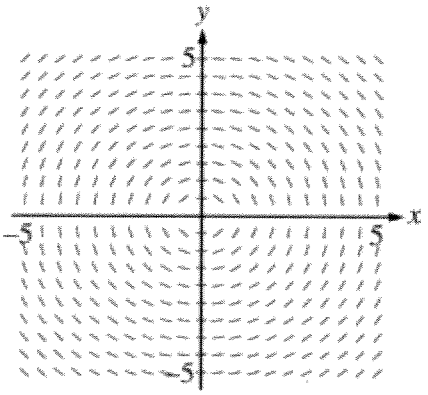
(B)



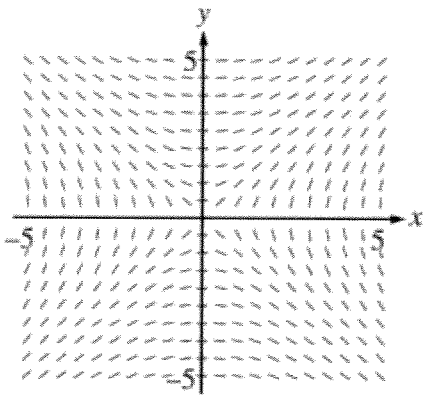
(C)



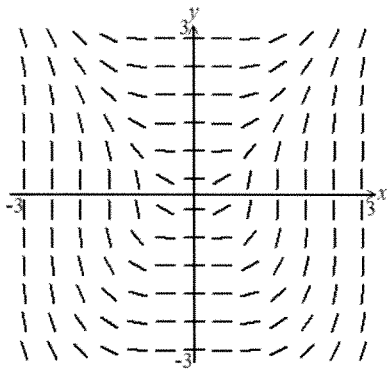
(D)



(E)



4. (calculator not allowed)



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

5. (calculator not allowed)
Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3 \ln 3}{\ln 2}$

(B) $\frac{2 \ln 3}{\ln 2}$

(C) $\frac{\ln 3}{\ln 2}$

(D) $\ln\left(\frac{27}{2}\right)$

(E) $\ln\left(\frac{9}{2}\right)$

6. (calculator not allowed)
If $\frac{dy}{dt} = -2y$ and if $y = -1$ when $t = 0$, what is the value of t for which $y = -\frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$

(B) $-\frac{1}{4}$

(C) $\frac{\ln 2}{2}$

(D) $\frac{\sqrt{2}}{2}$

(E) $\ln 2$

7. (calculator not allowed)
If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$

(B) $-\frac{1}{3}$

(C) 0

(D) $\frac{1}{3}$

(E) $\frac{2}{3}$

8. (calculator not allowed)
At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$
(B) $y = x^3 + 8$
(C) $y = e^{x^3} + 7$
(D) $y = \ln(x+1) + 8$
(E) $y^2 = x^3 + 8$

9. (calculator not allowed) If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3 + e^{-x^2}$
(B) $\sqrt{3} + e^{-x}$
(C) $1 + e^{-x}$
(D) $\sqrt{3 + e^{-x^2}}$
(E) $\sqrt{3 + e^{x^2}}$

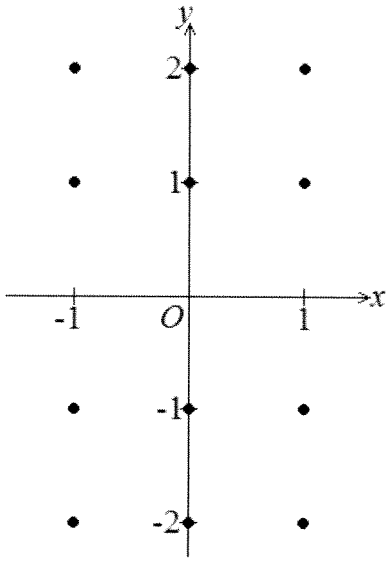
10. (calculator not allowed)
If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$
(B) $\sec^2 x + C$
(C) $\ln|\sec x| + C$
(D) $\ln|\cos x| + C$
(E) $\sec x \tan x + C$

Free Response

11. (calculator not allowed)

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

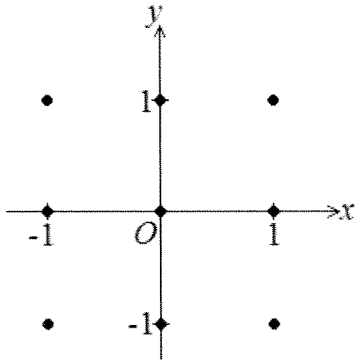
(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

12. (calculator not allowed)

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

13. (calculator not allowed)

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.