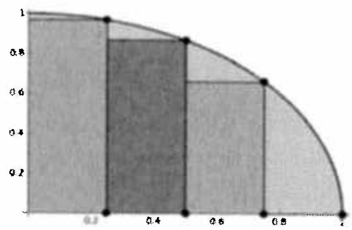
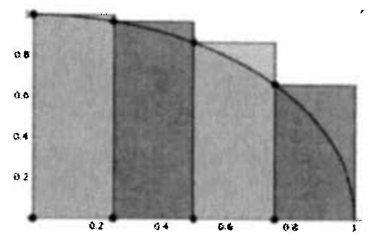
Accumulation

Left and right Riemann sums



Right Riemann Sum



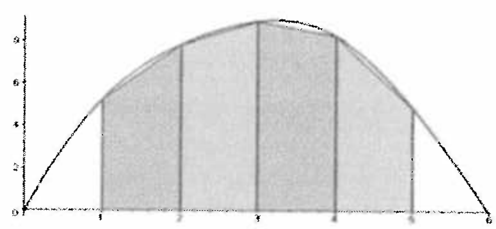
Left Riemann Sum

Correct justification for over and under approximations:

f(x)	Left Riemann Sum	Right Riemann Sum
Increasing ($f'(x) > 0$)	Under approximates the area because *f*(x) is increasing	Over approximates the area because *f*(x) is increasing
Decreasing ($f'(x) < 0$)	Over approximates the area because *f*(x) is decreasing	Under approximates the area because *f*(x) is decreasing

Incorrect Reasoning: *The left Riemann Sum is an under approximation because the rectangles are all underneath or below the graph. Stating that the rectangles are below the function is not acceptable mathematical reasoning. It merely restates that it is an under approximation but does not explain WHY.*

Trapezoidal approximations



Over/Under Approximations with Trapezoidal Approximations

f(x)	Trapezoidal Sum
Concave Up ($f''(x) > 0$)	Over approximates the area because $f''(x) > 0$
Concave Down ($f''(x) < 0$)	Under approximates the area because $f''(x) < 0$

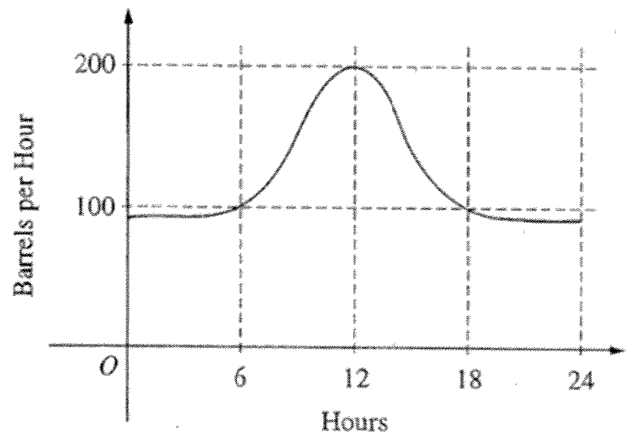
Student should be able to:

- Approximate the value of an integral using Riemann sums or trapezoidal sums and explain the meaning of the value in context.
- Evaluate a definite integral and explain the meaning of the answer in context
  - Recognize that a definite integral gives an accumulation or total
  - Always give meaning to the integral in CONTEXT to the problem
  - Give the units of measurement
  - Reference the limits of integration with appropriate units in the context of the problem
- Apply the fundamental theorems of calculus (sometimes using initial conditions) and explain the meaning of the answer in context
- Calculate derivatives based on results of previous calculations involving equations and approximations
- Solve application problems involving $Amount = \int_{begining\ time}^{ending\ time} Rate\ dt$ or

$$Current\ Amount = Initial\ Amount + \int_{time1}^{time2} addition\ rate\ dt - \int_{time1}^{time2} subtraction\ rate\ dt$$

Multiple Choice

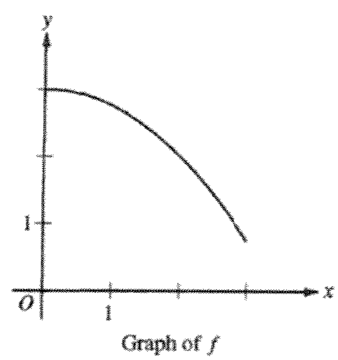
1. (calculator not allowed)



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

2. (calculator not allowed)



The graph of the function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

3. (calculator not allowed)
If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of

$\int_{-4}^{2} \frac{e^{-x}}{2}\, dx$?

(A) $e^2 + e^0 + e^{-2}$
(B) $e^4 + e^2 + e^0$
(C) $e^4 + 2e^2 + 2e^0 + e^{-2}$
(D) $\frac{1}{2}(e^4 + e^2 + e^0 + e^{-2})$
(E) $\frac{1}{2}(e^4 + 2e^2 + 2e^0 + e^{-2})$

4. (calculator allowed)

x	2	5	7	8
$f(x)$	10	30	40	20

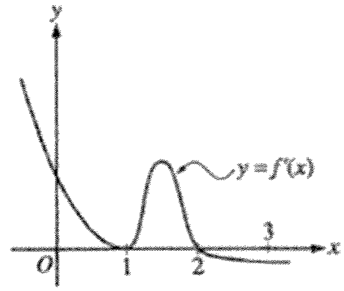
The function f, is continuous on the closed interval [2, 8] and has values that are given in the table above. Using the subintervals [2, 5], [5, 7], and [7, 8], what is the trapezoidal approximation of $\int_{2}^{8} f(x)\, dx$?

(A) 110
(B) 130
(C) 160
(D) 190
(E) 210

5. (calculator allowed)
If the definite integral $\int_0^2 e^{x^2}\, dx$ is first approximated by using a left Riemann sum with two subintervals of equal width and then approximated by using the trapezoidal rule with two equal subintervals, the difference between the two approximations is:

(A) 53.60
(B) 30.51
(C) 27.80
(D) 26.80
(E) 12.78

6. (calculator allowed)



The graph of f', the derivative of the function f, is shown above. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$

II. $f(2) > f(1)$

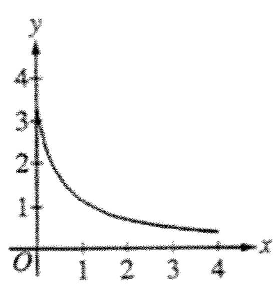
III. $f(1) > f(3)$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) II and III only

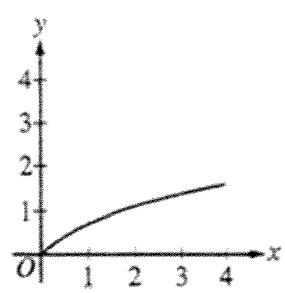
7. (calculator allowed)

If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

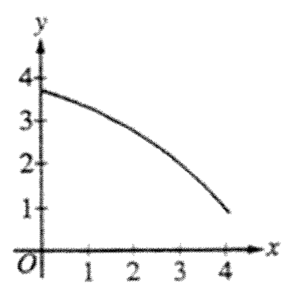
(A)



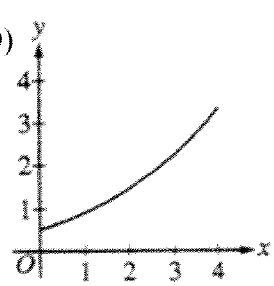
(B)



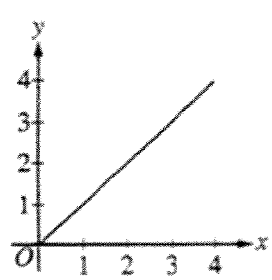
(C)



(D)



(E)



8. (calculator allowed)

t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec
(B) 30 ft/sec
(C) 7 ft/sec
(D) 39 ft/sec
(E) 41 ft/sec

Free Response

9. (calculator not allowed)

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

(Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

10. (calculator not allowed)
Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times t.

t *(seconds)*	0	10	40	60
$B(t)$ *(meters)*	100	136	9	49
$v(t)$ $\left(\frac{m}{\text{sec}}\right)$	2.0	2.3	2.5	4.6

(b) Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem.

Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

11. (calculator allowed)

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

12. (calculator allowed)

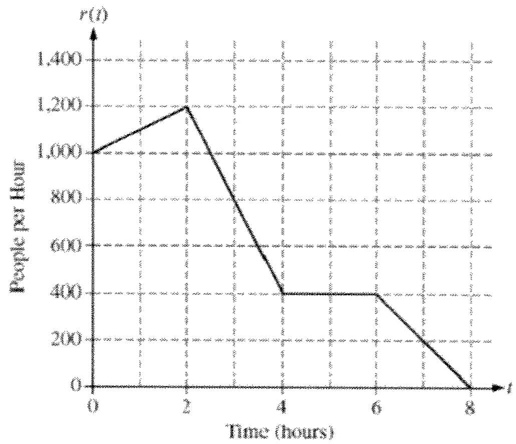
t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t=0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)\,dt$.

(c) Rocket B is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

13. (calculator allowed)



There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.