Related Rates

Suggested steps for solving related rate problems

1. Draw a picture of the situation described in the problem (if applicable).
2. List all given information and the quantities to be determined.
3. Label each quantity that varies with time with an appropriate variable.
4. Label each quantity that does not vary with its constant value.
5. Relate the variables in an equation and make appropriate substitutions.
6. Differentiate both sides of the equation with respect to time.
7. Substitute known quantities into the result and solve for the unknown quantity.
8. Include units with your answers.

Multiple Choice

1. (calculator not allowed)

   When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

   (A) 3
   (B) 4
   (C) 6
   (D) 8
   (E) 12

2. (calculator not allowed)

   The radius r of a sphere is increasing at the rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

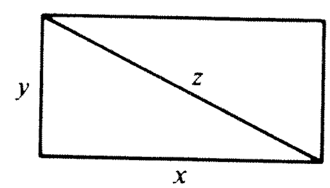
   (A) 10π
   (B) 12π
   (C) 22.5π
   (D) 25π
   (E) 30π

3. (calculator not allowed)

   The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

   (A) $\frac{1}{2}\pi$
   (B) 10π
   (C) 24π
   (D) 54π
   (E) 108π

4. (calculator not allowed)



   The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$, $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

   (A) $\frac{1}{3}$
   (B) 1
   (C) 2
   (D) $\sqrt{5}$
   (E) 5

5. (calculator not allowed)
A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meters per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$

(B) $\frac{4}{9}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

(E) $\frac{16}{9}$

6. (calculator not allowed)
Let $f(x)=\sqrt{x}$. If the rate of change of f at $x=c$ is twice its rate of change at $x=1$, then $c=$

(A) $\frac{1}{4}$

(B) 1

(C) 4

(D) $\frac{1}{\sqrt{2}}$

(E) $\frac{1}{2\sqrt{2}}$

7. (calculator not allowed)
If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

8. (calculator allowed)
The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$
(B) $-(0.1)C$
(C) $\frac{(0.1)C}{2\pi}$
(D) $(0.1)^2 C$
(E) $(0.1)^2 \pi C$

9. (calculator allowed)
A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60
(B) 57.88
(C) 59.20
(D) 60.00
(E) 67.40

10. (calculator allowed)
The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

(A) $0.04\pi\ \text{m}^2/\text{sec}$
(B) $0.4\pi\ \text{m}^2/\text{sec}$
(C) $4\pi\ \text{m}^2/\text{sec}$
(D) $20\pi\ \text{m}^2/\text{sec}$
(E) $100\pi\ \text{m}^2/\text{sec}$

Free Response

11. (calculator allowed)
Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeters, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of the change of the height of the oil slick with respect to the time, in centimeters per minute?

(b) A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

12. (calculator not allowed)

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

(Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

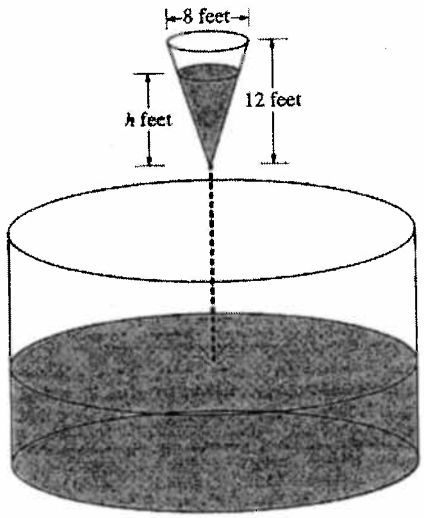
(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

13. (calculator allowed)

The wind chill is the temperature, in degrees Fahrenheit $(°F)$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32°F$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant $32°F$. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$? Indicate units of measure.

14. (calculator allowed)



As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute.

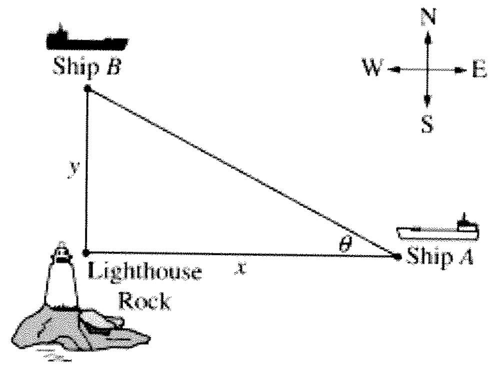
(The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate the units of measure.

15. (calculator not allowed)



Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.