Slope Fields and Differential Equations Solutions

We have intentionally included more material than can be covered in most Student Study Sessions to account for groups that are able to answer the questions at a faster rate. Use your own judgment, based on the group of students, to determine the order and selection of questions to work in the session. Be sure to include a variety of types of questions (multiple choice, free response, calculator, and non-calculator) in the time allotted.

Multiple Choice

1. C (1993 BC13 appropriate for AB)

$$\frac{dy}{y} = x^2\,dx$$
$$\int \frac{dy}{y} = \int x^2 dx$$
$$\ln|y| = \frac{1}{3}x^3 + C_1$$
$$e^{\frac{1}{3}x^3 + C_1} = y$$
$y = Ce^{\frac{1}{3}x^3}$; only answer C is of this form.

2. C (Course Description Exam Samples Calculus AB7)

$$y\;dy = 4x\;dx$$
$$\int y\;dy = \int 4x\;dx$$
$$\frac{y^2}{2} = \frac{4x^2}{2} + C$$
Use the initial condition $y(2) = -2$:
$$2 = 8 + C\;;\;C = -6$$
$$\frac{y^2}{2} = \frac{4x^2}{2} - 6$$
$$y^2 = 4x^2 - 12$$
$y = -\sqrt{4x^2 - 12}$ for $x > \sqrt{3}$

3. E (Course Description Exam Samples Calculus AB14)

Since $\frac{dy}{dx} = \frac{x}{y}$, anytime that $x = y$, the slope will be 1. The only graph that meets this condition for all quadrants is E.

4. E (2003 BC3 appropriate for AB)

$\frac{dy}{dx} > 0$ in quadrants I and IV, so B and E are viable graphs; however, $\frac{dy}{dx} < 0$ in quadrants II and III, so E is the only graph that meets both conditions.

5. A (1988 BC43 appropriate for AB)

$$\frac{dy}{dt} = ky$$
$$\int \frac{dy}{y} = \int k \, dt$$
$$\ln|y| = kt + C$$
$$e^{kt+C} = y$$
$$y = Ce^{kt}$$
$$2C = Ce^{3k}$$
$$2 = e^{3k}$$
$$\ln 2 = \ln e^{3k}$$
$$k = \frac{\ln 2}{3}$$
$$3C = Ce^{\frac{\ln 2}{3}t}$$
$$3 = e^{\frac{\ln 2}{3}t}$$
$$\ln 3 = \ln e^{\frac{\ln 2}{3}t}$$
$$t = \frac{3\ln 3}{\ln 2}$$

6. C (1985 BC33 appropriate for AB)

$$\frac{dy}{y} = -2 \, dt$$
$$\ln|y| = -2t + C$$
$$e^{-2t+C} = y$$
$$y = Ce^{-2t}$$

Using the initial condition $y = -1$ when $x = 1$:

$$C = -1$$
$$y = -e^{-2t}$$
$$-\frac{1}{2} = -1e^{-2t}$$
$$\frac{1}{2} = e^{-2t}$$
$$\ln\frac{1}{2} = \ln e^{-2t}$$
$$t = \frac{\ln\frac{1}{2}}{-2} = \frac{\ln 1 - \ln 2}{-2} = \frac{\ln 2}{2}$$

7. B (1993 AB33)

$$\frac{dy}{y^2} = 2\ dx$$

$$\int y^{-2}\ dy = \int 2\ dx$$

$$\frac{y^{-1}}{-1} = 2x + C$$

$$-\frac{1}{y} = 2x + C$$

Using the initial condition $y = -1$ when $x = 1$:

$$1 = 2 + C$$

$$C = -1$$

$$-\frac{1}{y} = 2x - 1$$

$$-\frac{1}{y} = 3$$

$$y = -\frac{1}{3}$$

8. A (1985 BC44 appropriate for AB)

$$\frac{dy}{dx} = 3x^2 y$$

$$\frac{dy}{y} = 3x^2 dx$$

$$\int \frac{dy}{y} = \int 3x^2 dx$$

$$\ln|y| = x^3 + C$$

$$e^{x^3 + C} = y$$

$$y = Ce^{x^3}$$

Using the point $(0, 8)$:

$$y = 8e^{x^3}$$

9. D (1969 BC23 appropriate for AB)

$$y\ dy = -xe^{-x^2}\,dx$$
$$\int y\ dy = \int -xe^{-x^2}\,dx$$
$$\frac{y^2}{2} = \frac{1}{2}e^{-x^2} + C$$
$$y^2 = e^{-x^2} + C$$

Using the given point $(0,\ 2)$:

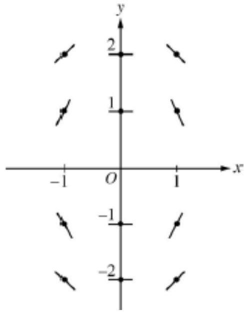
$$4 = 1 + C$$
$$C = 3$$
$$y^2 = e^{-x^2} + 3$$
$$y = \sqrt{3 + e^{-x^2}}$$

10. C (1969 AB27/BC27)

$$dy = \tan x\ dx$$
$$\int dy = \int \tan x\ dx$$
$$y = -\ln\left|\cos x\right| + C = \ln\left|\sec x\right| + C$$

Free Response
11. 2005 AB6

(a)



$2\begin{cases} 1: \text{ zero slopes} \\ 1: \text{ nonzero slopes} \end{cases}$

(b) The line tangent to f at $(1,-1)$ is

$y+1=2(x-1)$.

Thus, $f(1.1)$ is approximately -0.8.

$2\begin{cases} 1: \text{ equation of the tangent line} \\ 1: \text{ approximation for } f(1.1) \end{cases}$

(c) $\dfrac{dy}{dx}=-\dfrac{2x}{y}$

$y\,dy=-2x\,dx$

$\dfrac{y^2}{2}=-x^2+C$

$\dfrac{1}{2}=-1+C\,;C=\dfrac{3}{2}$

$y^2=-2x^2+3$

Since the particular solution goes through $(1,-1)$, y must be negative.

Thus the particular solution is $y=-\sqrt{3-2x^2}$.

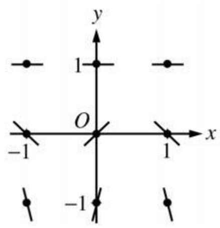
$5\begin{cases} 1: \text{ separates variables} \\ 1: \text{ antiderivatives} \\ 1: \text{ constant of integration} \\ 1: \text{ uses initial condition} \\ 1: \text{ solves for } y \end{cases}$

Note: $\max 2/5[1-1-0-0-0]$ if no constant of integration

Note: $0/5$ if no separation of variables

12. 2006 Form B AB5

(a)



2 { 1: zero slopes
1: all other slopes }

(b) The line $y=1$ satisfies the differential equation, so $c=1$.

1: $c=1$

(c) $\dfrac{1}{(y-1)^2}dy=\cos(\pi x)dx$

$-(y-1)^{-1}=\dfrac{1}{\pi}\sin(\pi x)+C$

$\dfrac{1}{1-y}=\dfrac{1}{\pi}\sin(\pi x)+C$

$1=\dfrac{1}{\pi}\sin(\pi)+C=C$

$\dfrac{1}{1-y}=\dfrac{1}{\pi}\sin(\pi x)+1$

$\dfrac{\pi}{1-y}=\sin(\pi x)+\pi$

$y=1-\dfrac{\pi}{\sin(\pi x)+x}$ for $-\infty<x<\infty$

6 { 1: separates variables
2: antiderivatives
1: constant of integration
1: uses initial condition
1: answer }

Note: max 3/6 [1-2-0-0-0] if no constant of integration
Note: 0/6 if no separation of variables

13. 2011 AB 5/BC5

(c) $\dfrac{dW}{dt} = \dfrac{1}{25}(W - 300)$

$$\int \frac{1}{W-300}\,dW = \int \frac{1}{25}\,dt$$

$$\ln|W - 300| = \frac{1}{25}t + C$$

$$\ln(1400 - 300) = \frac{1}{25}(0) + C \Rightarrow \ln(1100) = C$$

$$W - 300 = 1100e^{\frac{1}{25}t}$$

$$W(t) = 300 + 1100e^{\frac{1}{25}t},\ 0 \le t \le 20$$

$5\begin{cases} 1: & \text{separation of variables} \\ 1: & \text{antiderivatives} \\ 1: & \text{constant of integration} \\ 1: & \text{uses initial condition} \\ 1: & \text{solves for } W \end{cases}$

Note: max $2/5\,[1-1-0-0-0]$ if no constant of integration

Note: $1/5$ if no separation of variables