

Verify each identity. Use ONLY the Fundamental Identities. Show ALL Steps.

1. $\tan x + \cot x = \sec x \csc x$

2. $\frac{\tan \theta \cot \theta}{\csc \theta} = \sin \theta$

3. $\frac{1 - \cos x}{\sin x} = \csc x - \cot x$

4. $\cot \alpha + \frac{\sin \alpha}{1 + \cos \alpha} = \csc \alpha$

5. $\frac{\cos x}{1 - \sin x} + \frac{2 - \sin x}{\cos x} = 2 \sec x$

6. $1 - \frac{\sin^2 x}{1 + \cos x} = \cos x$

7. $\sin^2 \theta + \sin^2 \theta \cot^2 \theta = 1$

8. $(\tan^2 x + 1)(\cos^2 x + 1) = \tan^2 \theta + 2$

9. $3\sin^2 \theta + 4\cos^2 \theta = 3 + \cos^2 \theta$

10. $\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x} = 2 \tan x$

*For the problems below, leave answers in simplest radical or fractional form.

NO DECIMAL ANSWERS!!

I. Sum or Difference Identities.

A. Find the exact value of each trig value by rewriting the angle as the sum or difference of 2 special angles.

$\frac{\pi}{6} = \frac{2}{12}\pi$

$\frac{5}{12}\pi = \frac{2}{12}\pi + \frac{3}{12}\pi$

$\frac{\pi}{4} = \frac{3}{12}\pi$

1. $\cos \frac{5\pi}{12} = \cos \left(\frac{\pi}{6} + \frac{\pi}{4} \right)$

$\frac{\pi}{3} = \frac{4}{12}\pi$

$\cos \frac{\pi}{6} \cos \frac{\pi}{4} - \sin \frac{\pi}{6} \sin \frac{\pi}{4}$

$\left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

2. $\sin 75^\circ = \sin (45^\circ + 30^\circ)$

$\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$\left(\frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{3}}{2} \right) + \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{2} \right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

B. Find the exact value of each expression by simplifying to a special angle.

$\frac{\pi}{4} = \frac{3}{12}\pi$

3. $\frac{\tan 40^\circ + \tan 20^\circ}{1 - \tan 40^\circ \tan 20^\circ} = \tan (40^\circ + 20^\circ)$

$\tan 60^\circ$

$\boxed{\sqrt{3}}$

4. $\cos \frac{5\pi}{12} \cos \frac{\pi}{4} + \sin \frac{5\pi}{12} \sin \frac{\pi}{4}$

$\cos \left(\frac{5}{12}\pi - \frac{3}{12}\pi \right)$

$\cos \left(\frac{2}{12}\pi \right)$

$\cos \frac{\pi}{6} = \boxed{\frac{\sqrt{3}}{2}}$

II. Double and Half Angle Identities.

A. Find the exact value of each trig ratio by rewriting the angle as half of a special angle.

7. $\sin 22.5^\circ$

$$\begin{aligned} & \sin \frac{1}{2}(45^\circ) \\ &= + \sqrt{\frac{1 - \cos 45^\circ}{2}} \\ &= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} \cdot \frac{(2)}{(2)} = \sqrt{\frac{2 - \sqrt{2}}{4}} \end{aligned}$$

$$\boxed{\frac{\sqrt{2 - \sqrt{2}}}{2}}$$

8. $\cos 75^\circ = \cos \frac{1}{2}(150^\circ)$

$$\begin{aligned} &= + \sqrt{\frac{1 + \cos 150^\circ}{2}} \\ &= \sqrt{\frac{1 + (-\frac{\sqrt{3}}{2})}{2}} \cdot \frac{2}{2} \\ &= \sqrt{\frac{2 - \sqrt{3}}{2}} \end{aligned}$$

$$\boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$$

9. $\tan \frac{3}{8}\pi$

$$\begin{aligned} & \tan \frac{1}{2}(\frac{3}{4}\pi) \\ &= \frac{1 - \cos \frac{3}{4}\pi}{\sin \frac{3}{4}\pi} \\ &= \frac{1 - (-\frac{\sqrt{2}}{2})}{\frac{\sqrt{2}}{2}} \cdot \frac{2}{2} \\ &= \frac{2 + \sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2} + 2}{2} \end{aligned}$$

$$2 \cdot \frac{3}{8}\pi$$

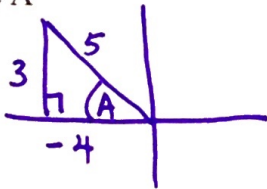
$$\frac{3}{4}\pi$$

$$\boxed{\sqrt{2} + 1}$$

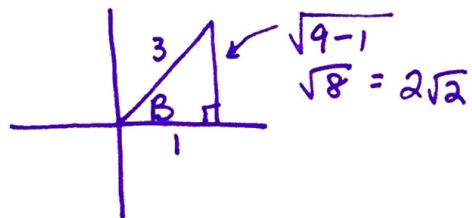
B. Given: $\tan A = -\frac{3}{4}$, $\frac{\pi}{2} < A < \pi$ and $\cos B = \frac{1}{3}$, $0 < B < \frac{\pi}{2}$

Set up a triangle in the correct quadrant for angle A and angle B. Use the triangle to find each value.

Angle A



Angle B



10. $\sin(A - B)$

$$\sin A \cos B - \cos A \sin B$$

$$\left(\frac{3}{5}\right)\left(\frac{1}{3}\right) - \left(-\frac{4}{5}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

$$\frac{3}{15} + \frac{8\sqrt{2}}{15}$$

$$\boxed{\frac{3 + 8\sqrt{2}}{15}}$$

11. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\frac{-\frac{3}{4} + 2\sqrt{2}}{1 - \left(-\frac{3}{4}\right)(2\sqrt{2})} \cdot \frac{(4)}{(4)}$$

$$\boxed{\frac{-3 + 8\sqrt{2}}{4 + 6\sqrt{2}}}$$

12. $\sin 2B$

$$2 \sin B \cos B$$

$$2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

$$\boxed{\frac{4\sqrt{2}}{9}}$$

13. $\cos \frac{A}{2} = \cos \frac{1}{2}A$

$$= + \sqrt{\frac{1 + \cos A}{2}}$$

$$\sqrt{\frac{1 + \left(-\frac{4}{5}\right)}{2}} \cdot \frac{5}{5}$$

$$\sqrt{\frac{5 - 4}{10}} = \frac{1}{\sqrt{10}} \rightarrow \boxed{\frac{\sqrt{10}}{10}}$$

$$\frac{1}{2}\left(\frac{\pi}{2}\right) < A < \left(\frac{\pi}{2}\right)$$

$$\frac{\pi}{4} < \frac{1}{2}A < \frac{\pi}{2}$$

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