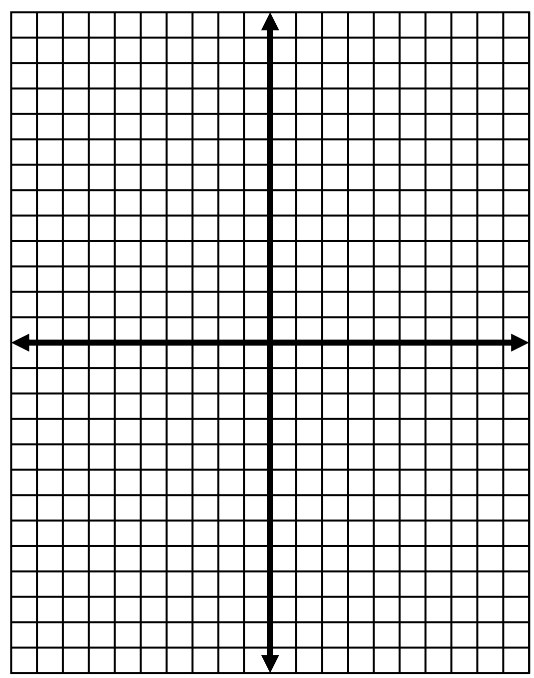
Rates of Change and Tangent Lines

For the function: $f(x) = x^2 - 3x + 5$:

1. Find the average rate of change on the interval $[-1\,,3]$.	2. Find the instantaneous rate of change at $x = 3$.
3. Find the y-value when $x = 3$.	4. Write the general point-slope form for the equation of a line.

5. Using $x = 3$, the y-coordinate found in #3, and the slope found in #2, write an equation for this line in point slope form.

6. Graph the function $f(x)$ (red) and the line from #5. (blue)
This blue line is the Tangent Line at $x = 3$.



7. Write the equation of the line that is perpendicular to the line in #5 at $x = 3$
Graph and label this the Normal Line on the grid. (green)

8. Write the equation of the line that passes through the points $(-1\,,9)$ and $(3,5)$.
Graph and label this line the Secant Line on the grid. (orange)

The line represented in #5 (blue) is called the **tangent line** to the function $f(x)\ at\ x = a$. The tangent line to a curve at a point $x = a$ (the point of tangency) is the line that intersects the curve at $x = a$ and has the same slope (instantaneous rate of change) as the curve at the point $x = a$.

General Equation:

$$\boldsymbol{y - f(a) = m(x - a)}$$

where $\boldsymbol{m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$

The line represented in #7 (green) is called the **normal line** to the function f(x) at $x = a$. The normal line to a curve at a point $x = a$ is a line that is perpendicular to a tangent line and goes through the point of tangency.

The slope of the normal line is the negative reciprocal of the slope of the tangent line.

The line represented in #8 (orange) is called a **secant line** to the function f(x). A secant line contains 2 points on the function and its slope is the average rate of change between those 2 points.

The slope of the secant line from point *a* to point *b* is: $\boldsymbol{m = \frac{f(b) - f(a)}{b - a}}$

Practice: $f(x) = x^3$

1. Write the equation of the line secant to the curve through $x = 0$ and $x = 2$.

2. Write the equation of the line tangent to the curve at $x = -3$

3. Write the equation of the line normal to the curve at $x = 1$.