2017 AP Calculus AB Sample Questions ***DO NOT WRITE ON WS Please!***

Section I: Part A (MC_No Calculator)

1. $\lim\limits_{x\to\pi} \dfrac{\cos x + \sin(2x) + 1}{x^2 - \pi^2}$ is

(A) $\dfrac{1}{2\pi}$

(B) $\dfrac{1}{\pi}$

(C) 1

(D) nonexistent

$\lim\limits_{x\to\pi} \dfrac{-\sin x + 2\cos(2x)}{2x}$

$\dfrac{-\sin\pi + 2\cos(2\pi)}{2\pi}$

$\dfrac{0+2(1)}{2\pi} = \boxed{\dfrac{1}{\pi}}$

EK 1.1C3: Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.

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2. $\lim\limits_{x\to\infty} \dfrac{\sqrt{9x^4+1}}{x^2 - 3x + 5}$ is

(A) 1

(B) 3

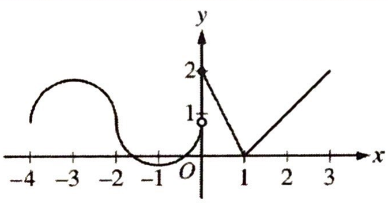
(C) 9

(D) nonexistent

$\dfrac{3\cancel{x^2}}{\cancel{x^2}}$

EK 1.1C2: The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.

EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.



Graph of f

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

EK 2.2B1: A continuous function may fail to be differentiable at a point in its domain.

EK 1.2A3: Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.

3. The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

$x = -2,\ x = 1$

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4. An ice sculpture in the form of a sphere melts in such a way that it maintains its spherical shape. The volume of the sphere is decreasing at a constant rate of 2π cubic meters per hour. At what rate, in square meters per hour, is the surface area of the sphere decreasing at the moment when the radius is 5 meters? (Note: For a sphere of radius r, the surface area is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.)

4π

(A) $\dfrac{4\pi}{5}$

(B) 40π

(C) $80\pi^2$

(D) 100π

$\dfrac{dV}{dt} = 2\pi\ m^3/h$

$r = 5$

Find $\dfrac{dA}{dt}$?

EK 2.3C2: The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.

EK 2.1C5: The chain rule is the basis for implicit differentiation.

$V = \frac{4}{3}\pi r^3$

$\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$

$\dfrac{1}{4\pi r^2}\dfrac{dV}{dt} = \dfrac{dr}{dt}$

$A = 4\pi r^2$

$\dfrac{dA}{dt} = 8\pi r \dfrac{dr}{dt}$

$\dfrac{dA}{dt} = \cancel{8\pi r}\left(\dfrac{1}{\cancel{4\pi r^2}}\right)\dfrac{dV}{dt}$

$\dfrac{dA}{dt} = \dfrac{2}{r}\dfrac{dV}{dt}$

$\dfrac{dA}{dt} = \dfrac{2}{5}(2\pi)$

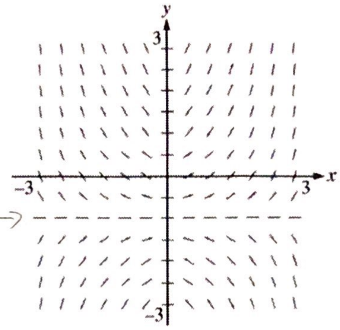
$\dfrac{dA}{dt} = \boxed{\dfrac{4\pi}{5}}$

5. Shown to the right is a slope field for which of the following differential equation?

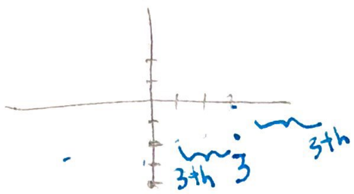
(A) $\dfrac{dy}{dx} = xy + x$ (C) $\dfrac{dy}{dx} = y + 1$

(B) $\dfrac{dy}{dx} = xy + y$ (D) $\dfrac{dy}{dx} = (x+1)^2$

$y = -2$



EK 2.3F1: Slope fields provide visual clues to the behavior of solutions to first order differential equations.



$$f(x) = \begin{cases} 2x - 2 & \text{for } x < 3 \\ 2x - 4 & \text{for } x \geq 3 \end{cases}$$

6. Let f be the piecewise-linear function defined above. Which of the following statements are true?

I. $\displaystyle\lim_{h \to 0^-} \frac{f(3+h) - f(3)}{h} = 2$ 4

II. $\displaystyle\lim_{h \to 0^+} \frac{f(3+h) - f(3)}{h} = 2$ ✓

III. $f'(3) = 2$ $f'(3) = \text{und}$

$f(x)$ is discontinuous at $x = 3$

(A) None

(B) II only

(C) I and II only

(D) I, II, and III

EK 2.1A2: The instantaneous rate of change of a function at a point can be expressed by

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.

EK 1.1A2: The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.

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7. If $f(x) = \displaystyle\int_1^{x^3} \frac{1}{1 + \ln t}\, dt$ for $x \geq 1$, then $f'(2) =$

$f'(x) = \frac{d}{dx}\int$

(A) $\dfrac{1}{1 + \ln 2}$ (C) $\dfrac{1}{1 + \ln 8}$

(B) $\dfrac{12}{1 + \ln 2}$ (D) $\dfrac{12}{1 + \ln 8}$

EK 3.3A2: If f is a continuous function on the interval $[a, b]$, then $\frac{d}{dx}\left(\int_a^x f(t)\, dt\right) = f(x)$, where x is between a and b.

EK 2.1C4: The chain rule provides a way to differentiate composite functions.

$f'(x) = \dfrac{1}{1 + \ln x^3}(3x^2)$

$f'(2) = \dfrac{3(2)^2}{1 + \ln 2^3} = \dfrac{12}{1 + \ln 8}$

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8. Which of the following limits is equal to $\displaystyle\int_3^5 x^4\, dx$?

$\Delta x = \dfrac{5-3}{n} = \dfrac{2}{n}$

(A) $\displaystyle\lim_{n \to \infty} \sum_{k=1}^{n} \left(3 + \frac{k}{n}\right)^4 \frac{1}{n}$ (C) $\displaystyle\lim_{n \to \infty} \sum_{k=1}^{n} \left(3 + \frac{2k}{n}\right)^4 \frac{1}{n}$

(B) $\displaystyle\lim_{n \to \infty} \sum_{k=1}^{n} \left(3 + \frac{k}{n}\right)^4 \frac{2}{n}$ (D) $\displaystyle\lim_{n \to \infty} \sum_{k=1}^{n} \left(3 + \frac{\boxed{2k}}{\boxed{n}}\right)^4 \boxed{\frac{2}{n}}$ Δx

EK 3.2A3: The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.

9. The function f is continuous for $-4 \le x \le 4$. The graph of f shown above consists of five line segments. What is the average value of f on the interval $-4 \le x \le 4$?

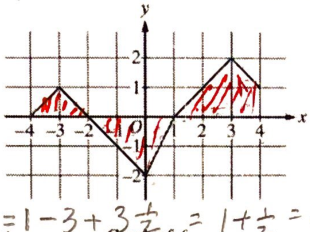
$\frac{1}{4-(-4)}\left(\frac{3}{2}\right) = \frac{1}{8}\cdot\frac{3}{2} = \boxed{\frac{3}{16}}$

(A) $\frac{1}{8}$

(B) $\frac{3}{16}$

(C) $\frac{15}{16}$

(D) $\frac{3}{2}$



$\int_{-4}^{4} f(x)\,dx = 1-3+3\frac{1}{2} = 1+\frac{1}{2} = \frac{3}{2}$

Graph of f

EK 3.4B1: The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

EK 3.2C1: In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.

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t	0	2
$f(t)$	4	12

Separate
Integrate

$\int \frac{1}{y}\,dy = \int k\,dt$

$e^{\ln y} = e^{kt+c}$

10. Let $y = f(t)$ be a solution to the differential equation $\frac{dy}{dt} = ky$, where k is a constant. Values of f for selected values of t are given in the table above. Which of the following is an expression for $f(t)$?

$y = e^{kt+c}$

$y = e^{kt}\cdot e^{c}$

$y = e^{kt}\cdot C$

$y = Ce^{kt}$

$f(t) = Ce^{kt}$

$12 = 4e^{2k}$

$3 = e^{2k}$

$\ln 3 = 2k$

$k = \frac{\ln 3}{2}$

$t=0$ $f(t)=4$

$4 = Ce^{0k}$

$4 = C$

(A) $4e^{\frac{t}{2}\ln 3}$

(B) $e^{\frac{t}{2}\ln 9} + 3$

(C) $2t^2 + 4$

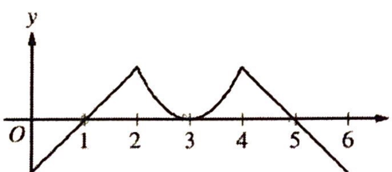
(D) $4t + 4$

EK 3.5B1: The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.

$4e^{\frac{\ln 3}{2}\cdot t}$ $\quad \frac{t\ln 3}{2} = \frac{t}{2}\ln 3$

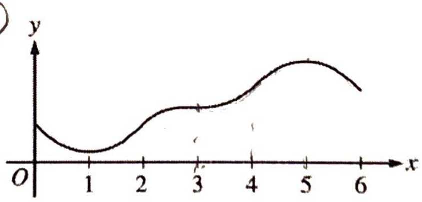
Section I: Part B (MC_ Calculator Active)

11. The graph of f', the derivative of the function f, is shown below. Which of the following could be the graph of f?

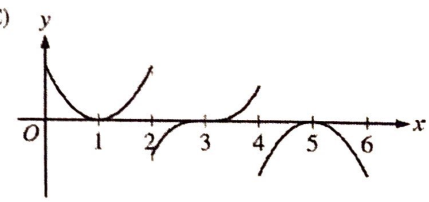


Graph of f'

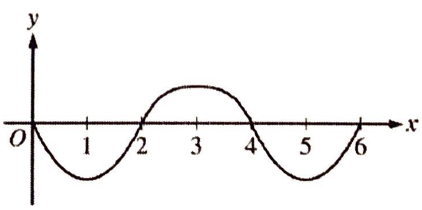
(A)



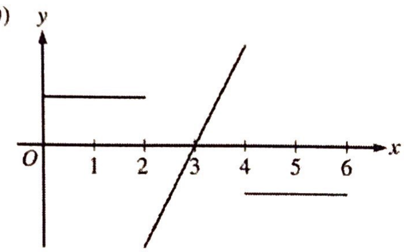
(C)



(B)



(D)



EK 2.2A3: Key features of the graphs of f, f', and f'' are related to one another.

EK 2.2B2: If a function is differentiable at a point, then it is continuous at that point.

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12. The derivative of a function f is given by $f'(x) = e^{\sin x} - \cos x - 1$ for $0 < x < 9$. On what intervals is f decreasing?

(A) $0 < x < 0.633$ and $4.115 < x < 6.916$

(B) $0 < x < 1.947$ and $5.744 < x < 8.230$

(C) $0.633 < x < 4.115$ and $6.916 < x < 9$

(D) $1.947 < x < 5.744$ and $8.230 < x < 9$

f decreasing where $f'(x) < 0$

EK 2.2A1: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.

13. The temperature of a room, in degrees Fahrenheit, is modeled by H, a differentiable function of the number of minutes after the thermostat is adjusted. Of the following, which is the best interpretation of $H'(5) = 2$?

(A) The temperature of the room is 2 degrees Fahrenheit, 5 minutes after the thermostat is adjusted.

(B) The temperature of the room increases by 2 degrees Fahrenheit during the first 5 minutes after the thermostat is adjusted.

(C) The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Fahrenheit per minute.

(D) The temperature of the room is increasing at a rate of 2 degrees Fahrenheit per minute, 5 minutes after the thermostat is adjusted.

EK 2.3A1: The unit for $f'(x)$ is the unit for f divided by the unit for x.

EK 2.3D1: The derivative can be used to express information about rates of change in applied contexts.

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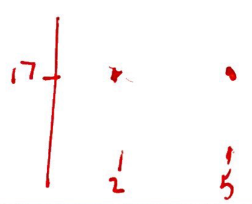
14. A function f is continuous on the closed interval $[2, 5]$ with $f(2) = 17$ and $f(5) = 17$. Which of the following additional conditions guarantees that there is a number c in the open interval $(2, 5)$ such that $f'(c) = 0$?

(A) No additional conditions are necessary.

(B) f has a relative extremum on the open interval $(2, 5)$.

(C) f is differentiable on the open interval $(2, 5)$.

(D) $\int_2^5 f(x)\,dx$ exists.



EK 2.4A1: If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b), the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

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15. A rain barrel collects water off the roof of a house during three hours of heavy rainfall. The height of the water in the barrel increases at the rate of $r(t) = 4t^3 e^{-1.5t}$ feet per hour, where t is the time in hours since the rain began. At time $t = 1$ hour, the height of the water is 0.75 foot. What is the height of the water in the barrel at time $t = 2$ hours?

(A) 1.361 ft

(B) 1.500 ft

(C) 1.672 ft

(D) 2.111 ft

$.75 + \int_1^2 r(t)\,dt$

EK 3.4E1: The definite integral can be used to express information about accumulation and net change in many applied contexts.

EK 3.3B2: If f is continuous on the interval $[a, b]$ and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

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16. A race car is traveling on a straight track at a velocity of 80 meters per second when the brakes are applied at time $t = 0$ seconds. From time $t = 0$ to the moment the race car stops, the acceleration of the race car is given by $a(t) = -6t^2 - t$ meters per second per second. During this time period, how far does the race car travel?

$x = 3.3386$

$\int_0 v(t)\,dt$

(A) 188.229 m

(B) 198.766 m

(C) 260.042 m

(D) 267.089 m

$v(0) = 80$

$v(t) = \int(-6t^2 - t)\,dt$

$v(t) = -6\left(\frac{t^3}{3}\right) - \frac{t^2}{2} + c$

$v(t) = -2t^3 - \frac{1}{2}t^2 + c$

$80 = c$

$v(t) = -2t^3 - \frac{1}{2}t^2 + 80$

find t when $v(t) = 0$ on Graph

store value

EK 3.4C1: For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.

EK 3.1A2: Differentiation rules provide the foundation for finding antiderivatives.