**AP Calculus – Plan of Action**

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| **When you see the words…..** | **This is your plan of action** |
| **1. Find the zeros of a function** | **Set the function equal to zero and solve for x** |
| **2. Find the domain of a function** | **Analyze the function by looking for radical expressions with even index (square roots, 4th roots, etc.). Exclude values of x that would yield a negative discriminant.**  **Also exclude values of x that would yield a denominator of zero.** |
| **3. Find the range of *f* (*x*) on [a, b]** | **If *f* is continuous on [a, b], then the range of *f* will be between [absolute min value of *f*, absolute max value of *f* ]** |
| **4. Show that is even** | **Evaluate *f* at x = - a and x = a and show they are equal** |
| **5. Show that is odd** | **Evaluate *f* at x = - a and x = a and show they are of opposite signs** |
| **6. Show that exists** | **Show that =** |
| **7. Evaluate** | **First use direct substitution. If the indeterminate form (0/0) is found, use factoring, rationalizing, or L'Hospital's Rule techniques** |
| **8. Show that is continuous at a point** | **Show that exists**  **Show that exists**  **i.e.**  **Finally, show that** |
| **9. Show that is differentiable at a point** | **Show that =** |
| **10. Find vertical asymptotes of** | **If *f* is a rational function, determine if it can be simplified further. Then the vertical asymptotes would be all values of x that would yield a zero denominator.** |
| **11. Find horizontal asymptotes of** | **Find . If these values exist, then the horizontal asymptotes are** |
| **12. Evaluate** | **If is a rational function, determine which of the numerator or denominator has the highest degree or if they are of equal degree.**  **If d > n, then the limit is equal to zero.**  **If d < n, then the limit does not exist ()**  **If d = n, then the limit is equal to the ratio of the coefficients of the highest degreed terms.**  **If is a transcendental function, then the limit behaves based upon the behavior of the dominate term.**  **Exponential functions dominate polynomial functions which dominate logarithmic functions.** |
| **13. Find the average rate of change of on [a, b]** | **This is the slope of the secant line between**  **\*\*Note: the question asks for the rate of change on an interval\*\*** |
| **14. Find the instantaneous rate of change of at x = a** | **This is another name for the derivative of the function evaluated at x = a, , or the slope of the tangent line to the curve at x = a.**  **\*\*Note: the question asks for the rate of change at a point\*\*** |
| **15. Find the derivative of by definition** |  |
| **16. Find** | **Use the Product Rule:** |
| **17. Find** | **Use the Quotient Rule:** |
| **18. Find the derivative of the composition function** | **Use the Chain Rule:** |
| **19. Estimate the value of given a table of values of on [*a, b*] where *c* is between *a* and *b*** | **Use two ordered pairs that are near *c* to evaluate** |
| **20. Find of an implicit function** | **Use Implicit Differentiation to take the derivative of every term with respect to x (don't forget to use the Chain Rule when taking the derivative of terms involving *y* – i.e. you'll multiply by a *y'* ). Then solve for *y'.*** |
| **21. Find of an implicit function** | **Take the derivative of using Implicit Differentiation. Then substitute for any *y'* with the expression you got for .** |
| **22. Find the derivative of an inverse function** | **If , then**  **.**  **Don't forget the domains and ranges of inverse functions "switch" ( if , then )** |
| **23. Find the equation of the line tangent to at x = a** | **Find which is the slope of the tangent line. Use the point-slope equation of a line to write:** |
| **24. Find the equation of the line normal to at x = a** | **Find which is the slope of the tangent line. The slope of the normal line is the negative reciprocal. Use point-slope to write:** |
| **25. Find where the tangent line to is horizontal.** | **Set and solve for x** |
| **26. Find where the tangent line to is vertical** | **Find values of x where is undefined due to a zero denominator.** |
| **27. Use a tangent line approximation to the graph of at *x = a* to estimate where *b* is a value close to *a.* Then determine if this approximation is an over or underestimate of the true value.** | **Find the equation of the tangent line to at *x = a.***    **Then substitute *b* in for *x* and simplify**    **If , then is concave up and the tangent line approximation is an underestimate.**  **If , then is concave down and the tangent line approximation is an overestimate.** |
| **28. Find critical values for a function** | **Find values of x where** |
| **29. Find the interval(s) where is increasing** | **Use a sign chart on to determine when .** |
| **30. Find the interval(s) where is decreasing** | **Use a sign chart on to determine when .** |
| **31. Find and classify any relative extrema of** | **Find critical numbers of . Use a sign chart on to determine when changes sign.**  ***f* has a relative maximum at x = c if changes from positive to negative.**  ***f* has a relative minimum at x = c if changes from negative to positive.** |
| **32. Find any inflection points of the graph of** | **Find values of x where changes sign.**  **\*\*Note: gives values of PIPs (Possible Inflection Points). There is only an inflection point if changes signs\*\***  **Given the graph of , the graph of will have an inflection point where has a relative extremum.** |
| **33. Find the interval(s) where the graph of is concave upwards** | **Use a sign chart on to determine when .**  **Given the graph of , the graph of will be concave upwards where increases.** |
| **34. Find the interval(s) where the graph of is concave downwards** | **Use a sign chart on to determine when .**  **Given the graph of , the graph of will be concave downwards where decreases.** |
| **35. Given either the equation or graph of and all values of critical numbers where , classify any relative extrema of .** | **Use the second derivative test to determine extrema:**  **If , then the graph of has a relative minimum at x = c.**  **If , then the graph of has a relative maximum at x = c.** |
| **36. For the continuous function *f*,find the value of *c,* if any, such that on the closed interval .** | **Intermediate Value Theorem (IVT):**  **Find . If , set and solve for x*.* Discard any value of x that is not in .**  **\*\*Special Case\*\***  **Find the value of *c*, if any, such that the continuous function *f* has a zero on**  **If have opposite signs, then there must exist at least one value of *c* in such that .**  **Set and solve for *x.* Discard any value of *x* that is not in .** |
| **37. For what value of *c,* if any, satisfies the Mean Value Theorem for on the closed interval .** | **Mean Value Theorem:**  **If a function is continuous on and differentiable on , then there exists at least one value of  *c* in such that:**    **\*Slope of a tangent line equals the slope of the secant line**  **\*Instantaneous rate of change equals the average rate of change** |
| **38. Find the absolute extrema of on a closed interval** | **Extreme Value Theorem (EVT):**  **If a function is continuous on , it must take on an absolute maximum and an absolute minimum value at least once on .**  **Absolute extrema may occur at critical numbers or at the endpoints of a closed interval. Use a "candidate's test" to determine the value of the function at any critical number on and the endpoints. The largest value is the absolute maximum and the smallest value is the absolute minimum.** |
| **39. Given , find .** | **Use an indefinite integral to find a 'family' of antiderivatives. Then use initial conditions to find C to 'solve' the differential equation.** |
| **40. Evaluate** | **Fundamental Theorem of Calculus Part 1:**  **If , then** |
| **41. Given , where u and v are functions of x, find** | **Fundamental Theorem of Calculus Part 2:**  **If , where u and v are functions of x, then**    **Ex:**  **Ex:** |
| **42. Describe the meaning of** | **This integral gives the net amount of change of from a to b.** |
| **43. Given the value of and that is the antiderivative of , find.** | **Use the FToC Part 1:** |
| **44. Find the average value of on** | **Average Value of a Function:**    **\*\*Note: Do not get this confused with average rate of change of a function.** |
| **45. The position of a particle moving along a straight line is given by , find the velocity and acceleration as functions of time.** |  |
| **46. Find the time(s) in which a particle is at rest** | **Find value(s) of *t* in which** |
| **47. Determine when a particle moving along a straight line changes direction** | **Use a sign chart on the velocity function and determine when the velocity changes signs.** |
| **48. Determine when a particle moves to the right/up/forward** | **Use a sign chart on the velocity function to find interval(s) in which** |
| **49. Determine when a particle moves to the left/down/backward** | **Use a sign chart on the velocity function to find interval(s) in which** |
| **50. Find when a particle is speeding up/slowing down** | **Use a sign chart on both the velocity and acceleration functions.**  **Speeding up when velocity and acceleration have the same signs.**  **Slowing down when velocity and acceleration have opposite signs.**  **\*\*Note: Common mistake is assuming that a particle is speeding up or slowing down when or respectively. This just tells us when the velocity is increasing or decreasing, but nothing about speed.** |
| **51. Find the speed of a particle** | **Speed =**  **\*\*The minimum possible speed is zero. However, the minimum velocity could be a negative number.** |
| **52. Find the average velocity of a particle on given the position function** | **Use the average rate of change equation** |
| **53. Find the average velocity of a particle on given the velocity function** | **Use the average value equation** |
| **54. Find the maximum/minimum velocity of a particle given the position function** | **To find the maximum/minimum velocity, we need to use a sign chart on the derivative of the velocity which is the acceleration.**  **If changes from positive to negative, then the velocity is a local maximum at this time.**  **If changes from negative to positive, then the velocity is a local minimum at this time.** |
| **55. Given the velocity of a particle moving along a straight line, find the change in position (also referred to displacement) on the time interval** | **Change in position =** |
| **56. Given the velocity of a particle moving along a straight line, find the total distance traveled on the time interval** | **Total distance traveled =** |
| **57. Given the velocity of a particle on and the initial position , find the position of the particle at time .** |  |
| **58. Given the velocity of a particle and the initial position, find the greatest distance from the origin of the particle on** | **Find the times in which the particle is at rest and the position function** .**Use the Extreme Value Theorem and evaluate at the endpoints and when the particle is at rest. Choose the greatest absolute value of .**  **\*\*This technique can also be used to determine when the particle is furthest right or left by choosing the greatest or least value of respectively.** |
| **59. Approximate the area bounded by and the x-axis on the interval using the following information about :**   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **x** | **0** | **3** | **6** | **10** | **14** | |  | **1** | **7** | **12** | **11** | **3** |   **(a) Use a left-hand Riemann sum with 4 subintervals**  **(b) Use a right-hand Riemann sum with 4 subintervals**  **(c) Use a midpoint Riemann sum with 2 subintervals**  **(d) Use a Trapezoidal sum with 4 trapezoids** | **(a)**  **(b)**  **(c)**  **(d)**  **OR** |
| **60. Find the area bounded by the graphs of .** | **Locate the intersections of by setting and use the values of x as the limits of integration if not already specified in the problem.**  **Determine which of the curves is above the other between the intersections points.** |
| **61. Find the line x = c that divides the area between the curve of on into two equal areas** | **Find a point c such that:** |
| **62. Find the volume of the solid created by rotating the region bounded by the curves of about the x-axis** | **Locate the intersections of by setting and use the values of x as the limits of integration if not already specified in the problem.**  **Determine which of the curves is above the other between the intersections points.** |
| **63. The region bounded by on forms the base of a solid. Find the volume of this solid whose cross section perpendicular to the x-axis is a …** | **Square:**  **Equilateral Triangle:**  **Semicircle:**  **General Cross Section:** |
| **64. Given a ski resort with an initial amount of snow of P, snow is added at the rate of S(t) and removed at the rate of R(t) on [t1, t2], find the rate the amount of snow is changing at *m*** | **If this is positive, then the amount of snow is increasing.**  **If this is negative, then the amount of snow is decreasing.** |
| **65. Given a ski resort with an initial amount of snow of P, snow is added at the rate of S(t) and removed at the rate of R(t) on [t1, t2], find the amount of snow at the resort after *m* minutes where t1 < *m* < t2** | **initial amount + net change of amount** |
| **66. Given a ski resort with an initial amount of snow of P, snow is added at the rate of S(t) and removed at the rate of R(t) on [t1, t2], find the time when the amount of snow is at a maximum/minimum.** | **Find the time(s) when . Then evaluate at these times and the endpoints t1 and t2 in the following equation.**  **(Application of the EVT)** |
| **67. Given , draw a slope field. Then sketch a particular solution given a point on the curve.** | **Identify points on the graph. Evaluate at these points. Draw a short line that represents the given slope at that point. The slope field should model the slope of a family of functions whose derivative is**  **Plot the initial point on the graph of the slope field and use the slope field to guide the 'flow' of the graph as you draw the particular solution to the left and right of the initial point.** |
| **68. Solve the differential equation and then find a particular solution given an initial point.** | **Separate the variables in the differential equation and then integrate each side remembering to include a constant of integration.**  **Use the initial point to find the constant of integration.**  **Finally, solve for y which is the particular solution.** |
| **69. Given the rate of change of a function that is increasing proportionally to the function itself, find the family of functions that describe the function as a function of time.** | **Separate the variables and integrate remembering to include a constant of integration.** |