

## Calculus Notes – Motion and Other Rates of Change

Recall: **Average** rate of change is the slope between **two** points.

**Instantaneous** rate of change is the slope at **one** point.

Given a position function  $s(t)$  or  $x(t)$ , where position is a function of time, the rate of change of position over time will give the velocity.

$$\text{Average velocity} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$$

$$\text{Instantaneous velocity} \\ v(t) = s'(t)$$

The rate of change of velocity over time will give the acceleration.

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\text{Instantaneous acceleration} \\ a(t) = v'(t) = s''(t)$$

Velocity and acceleration are **vectors**. They have magnitude and direction.

The **SIGN** of the **VELOCITY** tells the direction the object is moving.

$v(t) = 0$	$v(t) > 0$	$v(t) < 0$
Object is at Rest	Object is moving Right or Up	Object is moving Left or Down

**\*If velocity changes sign, the object changes direction.**

The **SIGN** of the **ACCELERATION** tells whether the velocity increases or decreases.

$a(t) > 0$ Object's velocity is Increasing	$a(t) < 0$ Object's velocity is Decreasing
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**SPEED** is a **scalar**. It has magnitude only (no direction).

$$\text{Speed } \textit{Speed} = |\textit{Velocity}|$$

**\*Speed is always positive!**

Speed Increases if velocity and acceleration have the same signs	Speed Decreases if velocity and acceleration have opposite signs
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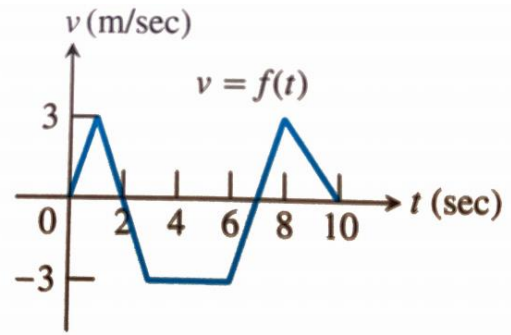
$$\text{Displacement} \\ \textit{Displacement} = \textit{Stop} - \textit{Start}$$

**\*Displacement can be positive or negative\***

$$\text{Distance traveled} \\ D_{\text{Total}} = |\textit{Turn} - \textit{Start}| + |\textit{Stop} - \textit{Turn}|$$

**\*Distance is always positive\***

Ex 1. The figure below shows the velocity  $v = \frac{ds}{dt} = f(t)$ ;  $\left(\frac{m}{sec}\right)$  of a body moving along a coordinate line. (problem #11 in Text section 3.4)



a. When does the body move to the right?

b. When does the body move to the left?

c. When is the body at rest?

d. When does the body reverse direction?

d. When is the body's acceleration positive?

e. When is the body's acceleration zero?

f. When is the body speeding up?

g. When is the body slowing down?

Ex 2. An object moves along a line so that its position at any time ( $t \geq 0$ ) is given by the function:

$$s(t) = \frac{1}{3}t^3 - 2t^2 + 3t$$

a. When is the object moving left?

b. When is the object at rest?

b. Find the displacement of the object from  $t = 0$  to  $t = 2$ .

c. Find the average velocity from  $t = 1$  to  $t = 3$ .

d. When is the object's velocity equal to 15?

e. Find the acceleration of the object when  $t = 2$

f. What is the speed of the object at  $t = 2$ ?