Worksheet 1. What You Need to Know About Motion Along the x-axis (Part 1)

In discussing motion, there are three closely related concepts that you need to keep straight. These are: ① $x(t)$ = position function
② $v(t) = x'(t)$ velocity function
③ $a(t) = v'(t) = x''(t)$ acceleration function

If $x(t)$ represents the position of a particle along the x-axis at any time t, then the following statements are true.

1. "Initially" means when t = 0.
2. "At the origin" means $x(0)$ = 0.
3. "At rest" means $x'(t)$ = 0.
4. If the velocity of the particle is positive, then the particle is moving to the up/right.
5. If the velocity of the particle is negative, then the particle is moving to the left.
6. To find average velocity over a time interval, divide the change in position by the change in time.
7. Instantaneous velocity is the velocity at a single moment (instant!) in time.
8. If the acceleration of the particle is positive, then the velocity is increasing.
9. If the acceleration of the particle is negative, then the velocity is decreasing.
10. In order for a particle to change direction, the velocity must change signs.
11. One way to determine total distance traveled over a time interval is to find the sum of the absolute values of the differences in position between all resting points. Here's an example: If the position of a particle is given by:

$$x(t) = \frac{1}{3}t^3 - t^2 - 3t + 4,$$

find the total distance traveled on the interval $0 \le t \le 6$.

$v(t) = x'(t) = t^2 - 2t - 3$

$0 = t^2 - 2t - 3$

$0 = (t-3)(t+1)$

$t = 3 \quad t = -1$

$x(0) = \frac{1}{3}(0)^3 - 0^2 - 3(0) + 4$
$= 4$

$x(3) = \frac{1}{3}(3)^3 - 3^2 - 3(3) + 4$
$= -5$

$x(6) = \frac{1}{3}(6)^3 - 6^2 - 3(6) + 4 = 22$

Dist $= |-5-4| + |22-(-5)|$
$= 9 + 27$
$= \boxed{36 \text{ units}}$

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Worksheet 2. Sample Practice Problems for the Topic of Motion (Part 1)

Example 1 (numerical).

The data in the table below give selected values for the velocity, in meters/minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.

Time t (min)	0	2	5	6	8	12
Velocity $v(t)$ (meters/min)	-3	2	3	5	7	5

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

particle is moving left, bc velocity is negative

2. Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Explain your answer.

Yes, sometime between 0 min and 2 min.
This is when velocity changes from negative to positive.
* By Intermediate Value Theorem, since velocity goes from negative to positive, it must go through zero and $V(t)=0 \rightarrow$ particle is at rest.

3. Use data from the table to find an approximation for $v'(10)$ and explain the meaning of $v'(10)$ in terms of the motion of the particle. Show the computations that lead to your answer and indicate units of measure.

Use difference quotient with the (closest) data given.
b.c. 10 lies between 8 and 12, Best approximation is given by:

$$v'(10) \approx \frac{V(12)-V(8)}{12-8} = \frac{5-7}{12-8} = -\frac{1}{2}\ \frac{m/min}{min} = -\frac{1}{2}\ m/m^2$$

$V'(10)$ is the acceleration of the particle at $t=10$

4. Let $a(t)$ denote the acceleration of the particle at time t. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify your answer.

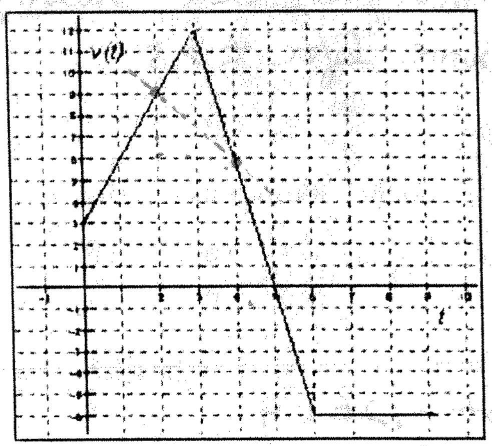
Yes, guaranteed by Mean Value Theorem or Rolle's Theorem. since velocity is differentiable (thus continuous) over ~~the~~ $6<t<12$

$$\frac{V(12)-V(6)}{12-6} = 0$$ then there must exist a point c in the interval such that $V'(c) = a(c) = 0$

$$\frac{5-5}{}$$

Example 2 (graphical).

The graph below represents the velocity v, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ to $t = 9$ seconds.



1. At $t = 4$ seconds, is the particle moving to the right or left? Explain your answer.

moving Rt b.c. $v'(t) > 0$

2. Over what time interval is the particle moving to the left? Explain your answer.

moving left over $(5, 9]$ b.c. velocity < 0

3. At $t = 4$ seconds, is the acceleration of the particle positive or negative? Explain your answer.

The acceleration is negative since velocity is decreasing. OR Acceleration is the slope of the velocity graph and the slope of the velocity graph at $t = 4$ is negative.

4. What is the average acceleration of the particle over the interval $2 \le t \le 4$? Show the computations that lead to your answer and indicate units of measure.

$$\frac{v(4)-v(2)}{4-2} = \frac{6-9}{4-2} = -\frac{3}{2}\ \frac{ft/sec}{sec} = -\frac{3}{2}\ \frac{ft}{sec^2}$$

5. Is there guaranteed to be a time t in the interval $2 \le t \le 4$ such that $v'(t) = -3/2$ ft/sec^2? Justify your answer.

NO guarantee. M.V.T does not apply $f(x)$ is not differentiable at $t = 3$.

6. At what time t in the given interval is the particle farthest to the right? Explain your answer. Particle is farthest to the right at $t=5$. velocity >0 over $0 \le t < 5$ moving Rt, then $v(t) < 0$, move left. $t > 5$

Example 3 (analytic).

A particle moves along the x-axis so that at time t its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.

$v(t) = x'(t) = 3t^2 - 12t + 9$

$x'(0) = 3(0)^2 - 12(0) + 9 = 9$

since $x'(t) > 0$, particle is moving Rt.

2. At $t = 1$, is the velocity of the particle increasing or decreasing? Explain your answer.

Acceleration

$a(t) = v'(t) = x''(t) = 6t - 12$

$a(1) = 6(1) - 12 = -6$ velocity is decreasing $a(1) < 0$

3. Find all values of t for which the particle is moving to the left.

find where $v(t) < 0$

		1		3	
$v(t)$	+	0	−	0	+
$(t-3)$	−		−		+
$(t-1)$	−		+		+

$v(t) = 3t^2 - 12t + 9$

$v(t) = 3(t^2 - 4t + 3) = 3(t-3)(t-1)$ $\boxed{1 < t < 3}$

4. Find the total distance traveled by the particle over the time interval $0 \le t \le 5$.

Distance Traveled $|x(1) - x(0)| + |x(3) - x(1)| + |x(5) - x(3)|$

$x(0) = 11$

$x(1) = 1^3 - 6(1)^2 + 9(1) + 11 = 15$

$x(3) = 3^3 - 6(3)^2 + 9(3) + 11 = 11$

$x(5) = 5^3 - 6(5)^2 + 9(5) + 11 = 31$

$= |15 - 11| + |11 - 15| + |31 - 11|$

$4 + 4 + 20$

$= 28$ units

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