Name:____Date:____________Block:______________

Finding Limits using Tables and Graphs

Limit Notation:

Suppose that *f* is a function defined on some interval containing the number *a*. The function *f* may or may not be defined at *a*. The limit notation $\lim_{x \to a} f(x) = L$ is read "the limit of *f(x)* as *x* approaches *a* equals the number L." This means that as *x* gets closer to *a*, but remains unequal to *a*, the corresponding values of *f(x)* get closer to L.

One-Sided Limits:

The limit notation $\lim_{x \to a^-} f(x) = L$ is read "the limit of *f(x)* as *x* approaches *a* from the left equals the number L" and is called the left-hand limit. This means that as *x* gets closer to *a*, but remains less than *a*, the corresponding values of *f(x)* get closer to L.

The limit notation $\lim_{x \to a^+} f(x) = L$ is read "the limit of *f(x)* as *x* approaches *a* from the right equals the number L" and is called the right-hand limit. This means that as *x* gets closer to *a*, but remains greater than *a*, the corresponding values of *f(x)* get closer to L.

Equal and Unequal One-Sided Limits:

One sided limits can be used to show that a function has a limit as x approaches a.

$\lim_{x \to a} f(x) = L$ if and only if both $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

One sided limits can be used to show that a function has no limit as x approaches a.

If $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = M$, where $L \neq M$, then $\lim_{x \to a} f(x)$ *does not exist.*

Finding Limits using a Table.

Use *x* values that approach *a* from the left and from the right. (use $a \pm .01$ *and* $a \pm .001$)

EX1. $\lim_{x \to 4} 3x^2$

	To the left of 4		4	To the right of 4	
x	3.99	3.999		4.001	4.01
$f(x)=3x^2$	$f(x) = 3(3.99)^2 = 47.7603$	$f(x) = 3(3.999)^2 = 47.9760$		$f(x) = 3(4.001)^2 = 48.0240$	$f(x) = 3(4.01)^2 = 48.2403$

EX2. $\lim_{x \to 3} 4x^2$

	To the left of			To the right of	
x					
f(x)=					

EX3. $\lim_{x \to 0} \frac{sinx}{x}$

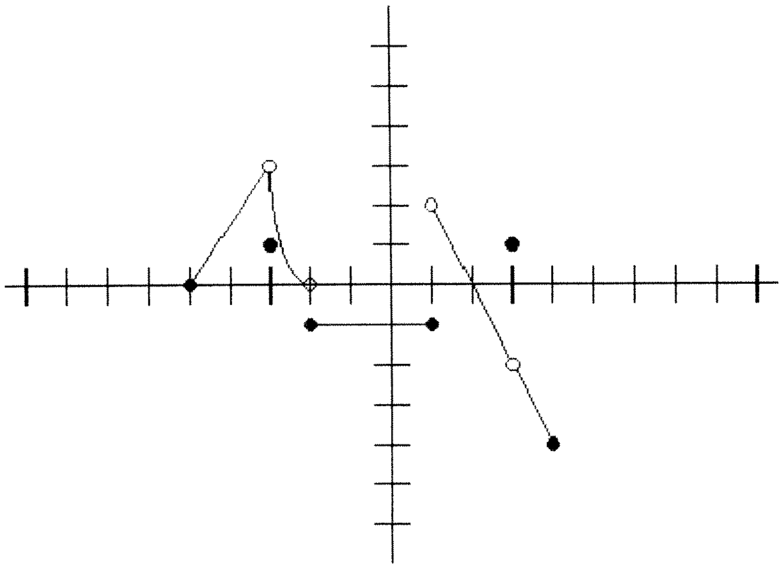
	To the left of			To the right of	
x					
f(x)=					

EX4. $\lim_{x \to 0} \frac{\cos x - 1}{x}$

	To the left of			To the right of	
x					
f(x)=					

Finding Limits Using Graphs

Let g be a function defined on the interval [-5,4] whose graph is given as:



Using the graph, find the following limits if they exist, and if not explain why not.

1.) $\lim_{x \to 3} g(x)$

2.) $\lim_{x \to 0} g(x)$

3.) $\lim_{x \to -3} g(x)$

4.) $\lim_{x \to 1^+} g(x)$

5.) $\lim_{x \to 1^-} g(x)$

6.) $\lim_{x \to 1} g(x)$

7.) $\lim_{x \to -2^-} g(x)$

8.) $\lim_{x \to 4} g(x)$

9.) $\lim_{x \to 2} g(x)$

10.) $\lim_{x \to -2^+} g(x)$

11.) f(-3)

12.) f(-2)

13.) f(1)

14.) f(3)

EX. Graph the function $f(x) = \begin{cases} 2x - 4 & if\ x \neq 3 \\ -5 & if\ x = 3 \end{cases}$

Use the graph to find $\lim_{x \to 3} f(x)$