Limit Exploration　　　　　　　　Calculus　　Key

Informal Definition of Limit:

- If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the **limit** of $f(x)$ as x approaches c is L. We write $\lim_{x \to c} f(x) = L$.

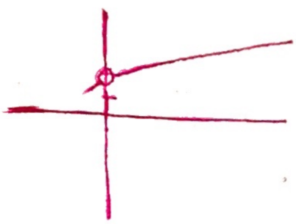
Find the Limit.

1. $\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1}$

Numerically

x	-0.001	-0.0001	0	0.0001	0.001
$f(x)$	1.9994998	1.99995		2.00005	2.0004999

Graphically



Analytically

*use conjugate $\sqrt{x+1}+1$

$(a+b)(a-b) = a^2-b^2$

$\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$

$\lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{(x+1)-1}$

$\lim_{x \to 0} \frac{\cancel{x}(\sqrt{x+1}+1)}{\cancel{x}} = \lim_{x \to 0} \sqrt{x+1}+1$

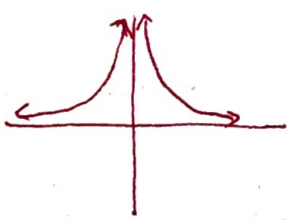
$\sqrt{0+1}+1 = \boxed{2}$

2. $\lim_{x \to 0} \frac{1}{x^2}$

Numerically

x	-0.001	-0.0001	0	0.0001	0.001
$f(x)$		Huge 100,000,000		Huge 100,000,000	

Graphically



Analytically

* Test points

Left　$x=0$　Right

$x = -.01$ | $x = .01$

$\frac{1}{(-.01)^2}$ | $\frac{1}{(.01)^2}$

$\frac{+}{+} = \infty$ | $\frac{+}{+} = \infty$

$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$ and $\lim_{x \to 0^+} \frac{1}{x^2} = \infty$

then

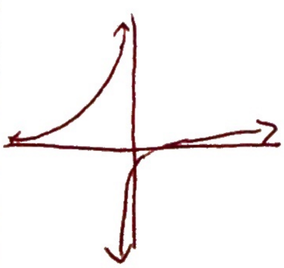
$\lim_{x \to 0} \frac{1}{x^2} = \infty$

3. $\lim_{x \to 2} \frac{2x^2-5x+2}{5x^2-7x-6}$

Numerically

x	1.999	1.9999	2	2.0001	2.001
$f(x)$	.230704	.2307627		.230775	.23083

Graphically



Analytically

$\lim_{x \to 2} \frac{\cancel{(x-2)}(2x-1)}{\cancel{(x-2)}(5x+3)}$

$\lim_{x \to 2} \frac{2x-1}{5x+3} = \frac{2(2)-1}{5(2)+3}$

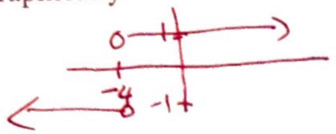
$\boxed{\frac{3}{13}}$

4. $\lim_{x\to-4}\frac{|x+4|}{x+4}$

Numerically

x	-4.001	-4.0001	-4	-3.9999	-3.999
$f(x)$	-1	-1		1	1

Graphically



$y=|x+4|$

$x+4=0$

$x=-4$

$x>-4$ $\quad y=x+4$

$x<-4$ $\quad y=-(x+4)$

Analytically

write $y=\frac{|x+4|}{x+4}$ as piece-wise

$y=\begin{cases}\frac{x+4}{x+4}, & x>-4\\ \frac{-(x+4)}{x+4}, & x<-4\end{cases}$ $\qquad y=\begin{cases}1, & x>-4\\ -1, & x<-4\end{cases}$

$\lim_{x\to-4^-}f(x)=-1$ and $\lim_{x\to-4^+}f(x)=1$

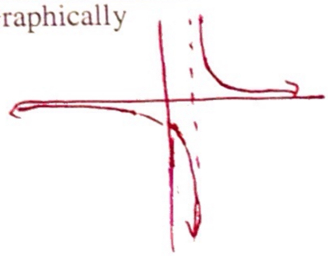
$\therefore \lim_{x\to-4}f(x)=DNE$

5. $\lim_{x\to0}\frac{\frac{1}{x-1}+1}{x}$

Numerically

x	-0.001	-0.0001	0	0.0001	0.001
$f(x)$	-.9990	-.9999		-1.0001	-1.0010

Graphically



Analytically

* Clear complex fraction by Top/Bottom by LCM

$\lim_{x\to0}\frac{\left(\frac{1}{x-1}+1\right)}{x}\cdot\frac{x-1}{x-1}$

$\lim_{x\to0}\frac{1+(x-1)}{x(x-1)}$

$\lim_{x\to0}\frac{\cancel{x}}{\cancel{x}(x-1)}=\lim_{x\to0}\frac{1}{x-1}$

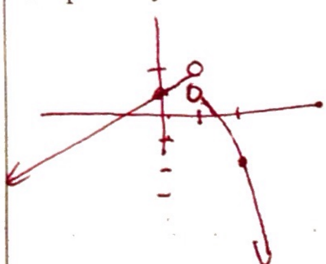
$=\frac{1}{0-1}=\boxed{-1}$

6. $\lim_{x\to1}f(x)$, where $f(x)=\begin{cases}x+1, & x<1\\ -x^2+2, & x>1\end{cases}$

Numerically

x	0.999	0.9999	1	1.0001	1.001
$f(x)$	1.999	1.9999		.999799	.997999

Graphically



Analytically

* one-sided limits

$\lim_{x\to1^-}f(x)=1+1=2$

$\lim_{x\to1^+}f(x)=-(1)^2+2$

$=1$

Since $\lim_{x\to1^-}f(x)\neq\lim_{x\to1^+}f(x)$

then $\lim_{x\to1}f(x)=\boxed{DNE}$

Helpful Limits:

$\lim_{x\to0}\frac{\sin x}{x}=1$ $\qquad$ $\lim_{x\to0}\frac{1-\cos x}{x}=0$

When substitution is used to find the limit, the indeterminate form of $\frac{0}{0}$ results. We will learn a technique call L'Hopital's Rule that can be used to find these limits. $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty} \textit{ are known as indeterminate form}\right)$