Limit Exploration Calculus

Informal Definition of Limit:

- If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the **limit** of $f(x)$ as x approaches c is L. We write $\lim_{x \to c} f(x) = L$.

Find the Limit.

1.

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1}$$

Numerically

x	-0.001	-0.0001	0	0.0001	0.001
$f(x)$					

Graphically

Analytically

2.

$$\lim_{x \to 0} \frac{1}{x^2}$$

Numerically

x	-0.001	-0.0001	0	0.0001	0.001
$f(x)$					

Graphically

Analytically

3.

$$\lim_{x \to 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$$

Numerically

x	1.999	1.9999	2	2.0001	2.001
$f(x)$					

Graphically

Analytically

4.

$$\lim_{x \to -4} \frac{|x+4|}{x+4}$$

Numerically

x	**-4.001**	**-4.0001**	**-4**	**-3.9999**	**-3.999**
$f(x)$					

Graphically

Analytically

5.

$$\lim_{x \to 0} \frac{\frac{1}{x-1} + 1}{x}$$

Numerically

x	**-0.001**	**-0.0001**	**0**	**0.0001**	**0.001**
$f(x)$					

Graphically

Analytically

6. $\lim_{x \to 1} f(x)$, where

$$f(x) = \begin{cases} x+1, & x < 0 \\ -x^2 + 3, & x > 0 \end{cases}$$

Numerically

x	**0.999**	**0.9999**	**1**	**1.0001**	**1.001**
$f(x)$					

Graphically

Analytically

Helpful Limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

When substitution is used to find the limit, the indeterminate form of $\frac{0}{0}$ results. We will learn a technique call L'Hopital's Rule that can be used to find these limits. $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty} \text{ are known as indeterminate form}\right)$