CALCULUS WORKSHEET – MOTION PROBLEMS

1.) A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t$.

a.) Show that at time t = 0, the particle is moving to the right.
b.) Find all values of t for which the particle is moving to the left.
c.) What is the position of the particle at time t = 3?
d.) When t = 3, what is the total distance the particle has traveled?

2.) A particle starts at time t = 0 and moves on a number line so that its position at time t is given by $s(t) = (t-2)^3(t-6)$, $v(t) = 4(t-2)^2(t-5)$, and $a(t) = 12(t-2)(t-4)$.

a.) When is the particle moving right?
b.) When is the particle at rest?
c.) When does the particle change direction?
d.) What is the farthest left the particle travels?
e.) When is the velocity decreasing?
f.) What is the total distance traveled from t = 0 to t = 6?
g.) What is the displacement from t = 0 to t = 6?

3.) The weight of many animals can be modeled by a Von Bertalanffy function $W(t) = a(1-be^{-ct})^3$ for some positive constants a, b, and c. For a population of female elephants, the weight, in kg, at age t, in yrs, is given by $W(t) = 2600(1-0.51e^{-0.075t})^3$.

* CALCULATOR *

a.) Show that W(t) is increasing for t > 0.
b.) Compute and interpret lim W(t).

c.) How fast is a newborn female elephant increasing in weight?
e.) An adult elephant weighs 1600kg. Determine her age.
f.) At what age is the rate of growth of an elephant a maximum?

4.) Let P(t) represent the price of a share of stock of a corporation at time t. Represent each of the following statements using derivatives and equations or inequalities.

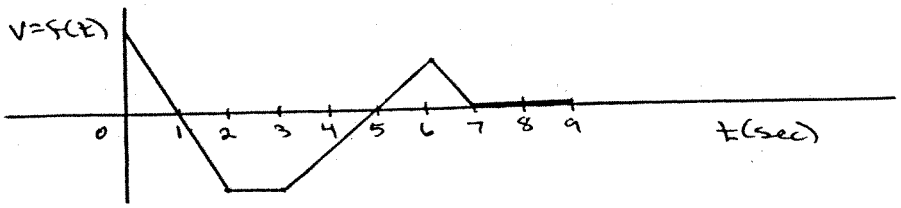
a.) The price of stock is rising faster and faster.
b.) The price of stock has bottomed out.

5.) The temperature, T, in degrees Fahrenheit, of a cold yam placed in a hot oven is given by T = f(t), where t is the time in minutes since the yam was put in the oven.

a.) What is the sign of f'(t)? Why?
b.) What are the units of f'(20)? What is the practical meaning of the statement f'(20) = 2?

6.) Suppose a particle is moving at varying velocity along a straight line and that s = f(t) represents the distance of the particle from a point as a function of time, t. Sketch a possible graph for f if the average velocity of the particle between t = 2 and t = 6 is the same as the instantaneous velocity at t = 5.

7.) The accompanying figure shows the velocity v = f(t) of a particle moving on a coordinate line.



a.) When does the particle move forward? move backward? speed up? slow down?
b.)When is the particle's acceleration positive? negative? zero?
c.)When does the particle move at its greatest speed?
d.)When does the particle stand still for more than an instant?

Answers:

1.) a.) v(0) = 8 (positive) b.) $2 < t < 4$ c.) 6 d.) $\frac{22}{3}$
2.) a.) $t > 5$ b.) $t = 2, t = 5$ c.) $t = 5$ d.) 27 units e.) $2 < t < 4$ f.) 102 units g.) -48
3.) a.) derivative always positive b.) 2600 kg c.) 71.634 kg/yr d.) t = 16.369 yrs
e.) t = 5.670 yrs
4.) a.) P''(t) > 0 b.) P'(t) = 0
5.) a.) f'(t) is positive because the temperature is increasing
b.) f'(20) in degrees/minute , at 20 minutes, the temperature of the yam is increasing 2 deg/min
6.) a.)forward[0,1),(5,7), backward (1,5), speed up(1,2),(5,6),slow down [0,1),(3,5),(6,7)
b.) pos (3,6), neg[0,2),(6,7), zero (2,3),(7,9]
c.) greatest speed t = 0, $2 < t < 3$
d.) stand still (7,9]