Graphical Relationships Among f, f', and f'' --Solutions

We have intentionally included more material than can be covered in most Student Study Sessions to account for groups that are able to answer the questions at a faster rate. Use your own judgment, based on the group of students, to determine the order and selection of questions to work in the session. Be sure to include a variety of types of questions (multiple choice, free response, calculator, and non-calculator) in the time allotted.

Multiple Choice:

1. B (1988 AB8)
   Since $\frac{dy}{dx} > 0$, $f(x)$ is increasing on the interval and since $\frac{d^2y}{dx^2} < 0$, the graph of $f(x)$ is concave down. Only the interval $b < x < c$ meets both of these criteria.

2. D (1998 AB17)
   $f(1) = 0$, $f'(1) > 0$ since f is increasing, and $f''(1) < 0$ since f is concave down, so answer D provides the correct order for the relationships between f and its derivatives at $x = 1$.

3. E (1969 AB30)
   Since f is continuous and using the points given, $-1 < x < 3$ while $-2 < y < 4$, the graph of f intersects both axes.

4. E (1997 AB11)
   Since $f' < 0$ for $x < -2$ and $x > 2$, the graph of f will be decreasing on these intervals. Since $f' > 0$ for $-2 < x < 2$, f is increasing on this interval. Since $f' = 0$ at $x = -2$ and changes from negative to positive, the graph of f has a local minimum at $x = -2$. Since $f' = 0$ at $x = 2$ and changes from positive to negative, the graph of f has a local maximum at $x = 2$.

5. A (2003 AB18)
   $g' \le 0$ on this interval

6. C (1985 BC20 appropriate for AB)
   $f(x)$ changes concavity three times on [2, 7].

7. E (1985 BC43 appropriate for AB)
   The only graph of f that is continuous, concave down, and matches the first derivative criteria is graph E.

8. E (1998 BC6 appropriate for AB)
   $h'(x) > 0$ where $h(x)$ is increasing and $h'(x) < 0$ where $h(x)$ is decreasing. Also, $h'(x) = 0$ where $h(x)$ has a relative maximum and a relative minimum..

9. E (1997 BC8 appropriate for AB)
   f' changes from increasing to decreasing or vice versa six times.

10. C (1997 BC7 appropriate for AB)
    Use the graph to determine that $f'(3) = 2.$ Use point slope to determine the equation in answer C

11. E (2003 AB28)
    Using the derivative information given, it can be determined that g is increasing at an increasing rate, therefore the only possible value for $g(6)$ using the answer choices is 27.

12. B (2003 AB90)
    Answer choices C,D, and E can be eliminated since those choices have a negative first derivative. Only answer B has $f(x)$ values increasing at a decreasing rate.

Free Response

13. 2000 AB3

(a) $x = -1$ $f'(x)$ changes from negative to positive at $x = -1$	$2\begin{cases}1: & \text{answer}\\1: & \text{justification}\end{cases}$
(b) $x = -5$ $f'(x)$ changes from positive to negative at $x = -5$	$2\begin{cases}1: & \text{answer}\\1: & \text{justification}\end{cases}$
(c) $f''(x)$ exists and f' is decreasing on the intervals $(-7, -3)$, $(2, 3)$, and $(3, 5)$	$2\begin{cases}1: & (-7, -3)\\1: & (2, 3)\cup(3, 5)\end{cases}$
.	

14. 2006B AB2

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

2 { 1: answer
1: reason }

15. 1991 AB5

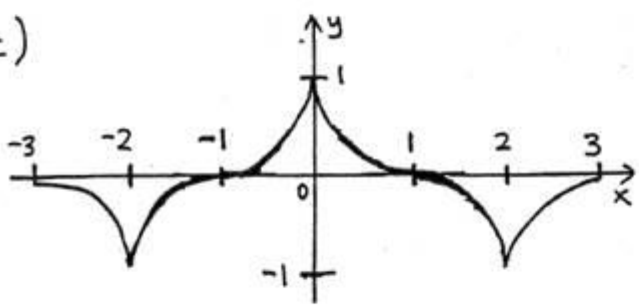
(a) Absolute maximum at $x = 0$
Absolute minimum at $x = \pm 2$

3 { +1: for each point correctly identified
-1: for each additional point or pair of symmetric points }

(b) Points of inflection at $x = \pm 1$ because the sign of $f''(x)$ changes at $x = 1$

3 { +1: for each point correctly identified
+1: justification (required on $[0, 3)$ only)
-1: for each additional point or pair of symmetric points }

(c)



3 { -1: for errors of:
- symmetry
- slopes on $[0, 3)$
- concavity on $[0, 3)$
- discontinuity on $[0, 3)$
- $f(0)$, $f(1)$, $f(2)$ incorrect of extra intercepts on $[0, 3)$

No more than 1 point can be lost in any one category }

16. 2004B AB4/BC4

(a)	$x=1$ and $x=3$ because the graph of f' changes from increasing to decreasing at $x=1$, and changes from decreasing to increasing at $x=3$.	2: { 1: $x=1$, $x=3$ 1: reason
(b)	The function f decreases from $x=-1$ to $x=4$, then increases from $x=4$ to $x=5$. Therefore, the absolute minimum value for f is at $x=4$. The absolute maximum value must occur at $x=-1$ or at $x=5$. $f(5)-f(-1)=\int_{-1}^{5} f'(t)\,dt<0$ Since $f(5)<f(-1)$, the absolute maximum value occurs at $x=-1$.	4: { 1: indicates f decreases then increases 1: eliminates $x=5$ for maximum 1: absolute minimum at $x=4$ 1: absolute maximum at $x=-1$