CALCULUS NOTES - RULES OF DIFFERENTIATION

Recall: $y = f(x)$ derivative $y' = f'(x) = \frac{dy}{dx} = \frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Rules:

1. The derivative of a constant is 0. $\frac{d}{dx}(c) = 0$

ex.) y = 3, y' = 0 **Think of the slope of a horizontal line!**

2. Power Rule: $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$, where n is a constant

ex.) $y = x^2$, $y' = 2x$
ex.) $y = x^7$, $y' = 7x^6$
ex.) $y = x$, $y' = 1$

3. Constant Multiplication Rule: $\frac{d}{dx}(cu) = c\frac{d}{dx}(u)$, where u is a function of x and c is a constant

ex.) $y = 3x^4$, $y' = 3\frac{d}{dx}\left(x^4\right) = 3 \cdot 4x^3 = 12x^3$

ex.) $y = 2x^9$, $y' = 2\frac{d}{dx}(x^9) = 2 \cdot 9x^8 = 18x^8$

4. Sum or Difference Rule: $\frac{d}{dx}(u \pm v) = \frac{d}{dx}(u) \pm \frac{d}{dx}(v)$, where u, v are functions of x

This rule can be extended to any number of terms.

ex.) $y = 9x^2 - 14x + 7$, $y' = \frac{d}{dx}(9x^2) - \frac{d}{dx}(14x) + \frac{d}{dx}(7) = 18x - 14$

ex.) $y = 6x^7 + 4x^5 - x$, $y' = \frac{d}{dx}(6x^7) + \frac{d}{dx}(4x^5) - \frac{d}{dx}(x) = 42x^6 + 20x^4 - 1$

5. Product Rule: $\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$, where u and v are functions of x

****Sing the product rule song!**** (to the tune of Mary Had A Little Lamb)

u d(v) plus v d(u)

Product Rule!

Product Rule!

u d(v) plus v d(u)

The product rule is cool!

ex.)

$$y = (x^2+1)(x^3+3),\quad y' = (x^2+1)\frac{d}{dx}(x^3+3) + (x^3+3)\frac{d}{dx}(x^2+1)$$
$$= (x^2+1)(3x^2) + (x^3+3)(2x)$$
$$= 3x^4 + 3x^2 + 2x^4 + 6x = 5x^4 + 3x^2 + 6x$$

ex.)

$$y = (3x^2-2x)(2x^3+x),\quad y' = (3x^2-2x)\frac{d}{dx}(2x^3+x) + (2x^3+x)\frac{d}{dx}(3x^2-2x)$$
$$= (3x^2-2x)(6x^2+1) + (2x^3+x)(6x-2)$$
$$= 18x^4 + 3x^2 - 12x^3 - 2x + 12x^4 - 4x^3 + 6x^2 - 2x$$
$$= 30x^4 - 16x^3 + 9x^2 - 4x$$

6. Quotient Rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$ OR $\frac{lo\ d\ hi - hi\ d\ lo}{lo\ lo}$

where u and v are functions of x

**** Sing the quotient rule song! **** (to the tune of Camptown Races)

lo d(hi) less hi d(lo)

lo lo, lo lo

The quotient rule's the way to go

To show how much we know!

ex.)

$$y=\frac{x^2+1}{x^2-1},\quad y'=\frac{\left(x^2-1\right)\frac{d}{dx}\left(x^2+1\right)-\left(x^2+1\right)\frac{d}{dx}\left(x^2-1\right)}{\left(x^2-1\right)^2}$$

$$=\frac{\left(x^2-1\right)\left(2x\right)-\left(x^2+1\right)\left(2x\right)}{\left(x^2-1\right)^2}$$

$$=\frac{2x^3-2x-2x^3-2x}{\left(x^2-1\right)^2}=\frac{-4x}{\left(x^2-1\right)^2}$$

ex.)

$$y=\frac{x^3}{2x-5},\quad y'=\frac{(2x-5)\frac{d}{dx}\left(x^3\right)-\left(x^3\right)\frac{d}{dx}(2x-5)}{(2x-5)^2}$$

$$=\frac{(2x-5)(3x^2)-\left(x^3\right)(2)}{(2x-5)^2}$$

$$=\frac{6x^3-15x^2-2x^3}{(2x-5)^2}=\frac{4x^3-15x^2}{(2x-5)^2}$$

ex.)

$y=\frac{2}{x^2}$, Rewrite using a negative exponent, then apply the power rule rather than the quotient rule.

$$y=2x^{-2},\quad y'=-4x^{-3}=\frac{-4}{x^3}$$

ex.)

$y=\frac{(2x+4)(x^3-x)}{x^2}$, Multiply out the numerator first.

$$y=\frac{2x^4+4x^3-2x^2-4x}{x^2}=2x^2+4x-2-4x^{-1}$$

$$y'=4x+4+4x^{-2}=4x+4+\frac{4}{x^2}$$

*** If f is even, then f' is odd. If f is odd, then f' is even. ***

You can also find second, third, and higher order derivatives.

$$y = x^4 - 3x^3 + 4x^2 + 9$$

$$y' = f'(x) = \frac{dy}{dx} = 4x^3 - 9x^2 + 8x$$

$$y'' = f''(x) = \frac{d^2y}{dx^2} = 12x^2 - 18x + 8$$

$$y''' = f'''(x) = \frac{d^3y}{dx^3} = 24x - 18$$

$$y^{iv} = f^4(x) = \frac{d^4y}{dx^4} = 24$$

$$y^v = f^5(x) = \frac{d^5y}{dx^5} = 0$$

All successive derivatives are equal to 0.

Derivatives can also be found in terms of other variables.

$A = \pi r^2, \quad \frac{dA}{dr} = 2\pi r$ This is the derivative of A with respect to r.

$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dr} = 4\pi r^2$ This is the derivative of V with respect to r.