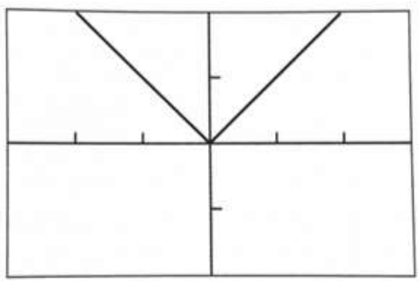
GN_Differentiability (3.2) Calculus AB Name_____________________

I. Where does a derivative fail to exist?

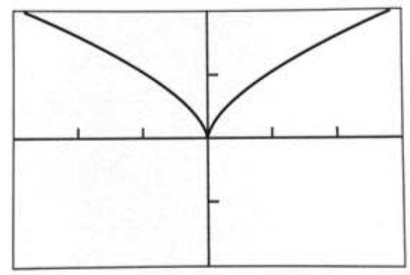
1. At any corner
(where the one-sided derivatives differ)



[−3, 3] by [−2, 2]

Figure 3.11 There is a "corner" at $x = 0$.

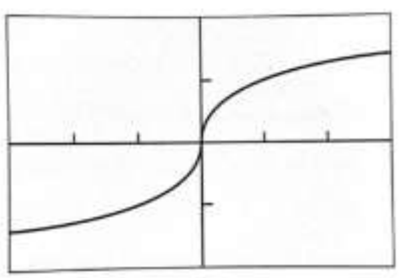
2. At any cusp
(where the slopes of the secant lines approach $-\infty$ from one side and ∞ from the other side)



[−3, 3] by [−2, 2]

Figure 3.12 There is a "cusp" at $x = 0$.

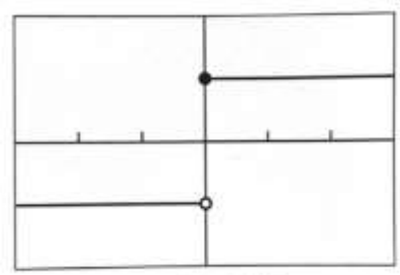
3. At any point where the tangent line is vertical
(where the slopes of the secant lines approach either $-\infty$ *or* ∞ from both sides)



[−3, 3] by [−2, 2]

Figure 3.13 There is a vertical tangent line at $x = 0$.

4. At any point of discontinuity
(this will cause one or both of the one-sided derivatives to be nonexistent)



[−3, 3] by [−2, 2]

Figure 3.14 There is a discontinuity at $x = 0$.

5. At an endpoint (A one-sided derivative may exist)

II. Derivatives of piece-wise functions.

Ex: $f(x) = \begin{cases} x^2 & if\ x \le 0 \\ 2x & if\ x > 0 \end{cases}$

III. Theorems

1. Differentiability implies LOCAL LINEARITY

A function that is differentiable at "$\boldsymbol{a}$" closely resembles its own tangent line very close to "$\boldsymbol{a}$". That is the curve will "straighten out" as we zoom in on it at a point of differentiability.

Is either of these functions differentiable at $= 0$?

a. $f(x) = |x|$

b. $g(x) = \sqrt{x^2 + .0001} - .01$

2. Differentiability Implies Continuity

If $\boldsymbol{f}$ has a derivative at $x = a$, then $\boldsymbol{f}$ is continuous at $x = a$.

Discontinuity implies non-differentiability

*****Continuity DOES NOT imply differentiability***** **ex:**$\boldsymbol{f(x) = |x|}$

3. Intermediate Value Theorem for Derivatives

If $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two points in an interval on which $\boldsymbol{f}$ is differentiable, then $\boldsymbol{f'}$ takes on every value between $\boldsymbol{f'(a) and\ f'(b)}$.

IV. Numerical Derivatives on a Calculator

On the calculator, use Math 8 function

*In older calculator, it will appear: nDeriv(function, x, value)

*In newer calculator, it will appear: $\frac{d}{d(\ \ )}(f(x))|_{(\ \ )=(\ \ )}$

Ex: Find the derivative of $y = x^3 - 3x + 3 \quad at\ x = 2$

*To graph a derivative with the original function in y_1 , use the Math 8 function in y_2.

$y_2 = nDeriv(y_1, x\ , x)$ or $y_2 = \frac{d}{dx}(y_2)|_{x=\,x}$