****FUNDAMENTAL TRIGONOMETRIC IDENTITIES**** Please **MEMORIZE** these!!

****Reciprocal Identities**		
$\sin x = \dfrac{1}{\csc x}$; $\csc x = \dfrac{1}{\sin x}$	$\cos x = \dfrac{1}{\sec x}$; $\sec x = \dfrac{1}{\cos x}$	$\tan x = \dfrac{1}{\cot x}$; $\cot x = \dfrac{1}{\tan x}$

****Quotient Identities**
$\tan x = \dfrac{\sin x}{\cos x}$
$\cot x = \dfrac{\cos x}{\sin x}$

****Even-Odd Identities**	
Even Functions $f(-x) = f(x)$	**Odd Functions** $f(-x) = -f(x)$
$\cos(-x) = \cos x$	$\sin(-x) = -\sin x$ $\tan(-x) = -\tan x$

****Pythagorean Identities**	****Other Forms of the Pythagorean Identities**	
$sin^2x + cos^2x = 1$ ➞	$1 - sin^2x = cos^2x$	$1 - cos^2x = sin^2x$
$1 + cot^2x = csc^2x$ ➞	$csc^2x - 1 = cot^2x$	$csc^2x - cot^2x = 1$
$tan^2x + 1 = sec^2x$ ➞	$sec^2x - 1 = tan^2x$	$sec^2x - tan^2x = 1$

****Double Angle**	
$\cos 2x = cos^2x - sin^2x$	$\sin 2x = 2sinx\cos x$
$\cos 2x = 2cos^2x - 1$	$\tan 2x = \dfrac{2\tan x}{1 - tan^2x}$
$\cos 2x = 1 - 2sin^2x$	

****Sum and Difference Identities**
$\sin(u+v) = \sin u\cos v + \cos u\sin v$ $\sin(u-v) = \sin u\cos v - \cos u\sin v$
$\cos(u+v) = cosu\cos v - \sin u\sin v$ $\cos(u-v) = \cos u\, cosv + \sin u\sin v$
$\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u\tan v}$ $\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u\tan v}$

****Half Angle**
$\sin\dfrac{x}{2} = \pm\sqrt{\dfrac{1-\cos x}{2}}$ *The sign of $\sin\frac{x}{2}$ depends on the quadrant in which $\frac{x}{2}$ lies.
$\cos\dfrac{x}{2} = \pm\sqrt{\dfrac{1+\cos x}{2}}$ The sign of $\cos\frac{x}{2}$ depends on the quadrant in which $\frac{x}{2}$ lies.
$\tan\dfrac{x}{2} = \dfrac{1-\cos x}{\sin x}$ or $\tan\dfrac{x}{2} = \dfrac{\sin x}{1+\cos x}$

HINTS on Proving or Verifying Trig Identities

*You must **MEMORIZE** the fundamental relationships between the functions. When you see one side of one of these identities, the other side should come to mind **IMMEDIATELY**.

*You are allowed to work down **one side** of the identity to transform it into the other side.
You may **NOT** work down both sides of the identity.

* Choose the more complex side then use identities and algebraic techniques to get it to look exactly like the other side

*As you work on one side of the identity, always keep an eye on the other side. It is easier to hit a target that you can see.
*YOU MAY NOT ADD, SUBTRACT, MULTIPLY, DIVIDE BOTH SIDES BY THE SAME THING. THIS IS NOT AN EQUATION THAT IS KNOWN TO BE EQUAL WHERE THAT IS ALLOWED. WE ARE VERIFYING THAT BOTH SIDES ARE EQUAL!!!

*Remember to follow the **RULES** of Algebra. (Yes, Algebra has RULES!)
Trigonometric fractions may be added, multiplied, and reduced just like algebraic fractions.

*Trigonometric expressions also may be factored and combined just like algebraic expressions.

*If one side of the equation contains only a single trigonometric function, eliminate all other functions if possible, from the other side.

*It is sometimes helpful to change all functions into sine and cosine functions before proceeding.

*If the numerator and denominator of a fraction contains a factor of "$1+\sin x$" or "$1-\sin x$", consider multiplying both the numerator and denominator by its **conjugate**. This operation creates a factor of "$\boldsymbol{1-sin^2x}$", which may be replaced by "$\boldsymbol{cos^2x}$".
The same idea applies to factors of "$\mathbf{1\pm\cos}\boldsymbol{x}$", "$\mathbf{sec}\,\boldsymbol{x}\pm\mathbf{1}$", "$\mathbf{sec}\,\boldsymbol{x}\pm\mathbf{tan}\,\boldsymbol{x}$",
"$\mathbf{csc}\,\boldsymbol{x}\pm\mathbf{1}$" and "$\mathbf{csc}\,\boldsymbol{x}\pm\mathbf{cot}\,\boldsymbol{x}$".

* Get a common denominator to combine 2 fractions into one.

*Rewrite a fraction as 2 terms. Note: $\dfrac{a+b}{c}=\dfrac{a}{c}+\dfrac{b}{c}$ $\quad \dfrac{a}{b+c}\neq!!\,\dfrac{a}{b}+\dfrac{a}{c}$

* Factoring pattern, fraction rules, ideas:

$$(a-b)(a+b)=a^2-b^2$$
$$(a\pm b)^2=a^2\pm 2ab+b^2$$
$$(a\pm b)(a^2\mp ab+b^2)=a^3\pm b^3$$

$$\frac{\frac{a+b}{c}}{\frac{d+e}{c}}=\frac{a+b}{d+e}$$

$$\frac{\frac{a+b}{c+d}}{e}=(a+b)\cdot\frac{e}{c+d}$$

***ALL** algebra **MUST** be shown. You **CAN NOT** say “I did it in my head.”

***<u>EACH step MUST be JUSTIFIED.</u>**
****<u>If I CANNOT see “HOW” you did it, you will NOT receive credit.</u>**

******BOTTOM LINE..DON’T just stare at the problem.***
******TRY ANYTHING and EVERYTHING!!!***