

Key

Drill_Rate of Change

1. The table below shows the distance an object traveled over a period of time.

| | | | | | | | |
|----------|----|----|----|----|----|----|----|
| t (sec) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| y (feet) | 10 | 45 | 70 | 85 | 90 | 85 | 70 |

a. Compute the **average rate of change** over the interval of [2, 5]. Include units in your answer.

$$\frac{f(5)-f(2)}{5-2} = \frac{85-70}{3} = \frac{15}{3} = \boxed{5}$$

b. Write the equation of the **secant line** over the interval of [2, 5].

$$m=5 \quad (2,70) \quad y-70=5(x-2) \quad \text{or} \quad (5,85) \quad y-85=5(x-5)$$

2. Given: $f(x) = 2x^2 - 5x$

a. Find the average rate of change of the function over the interval of [-1, 4].

$$f(4) = 2(4)^2 - 5(4) = 32 - 20 = 12$$

$$f(-1) = 2(-1)^2 - 5(-1) = 2 + 5 = 7$$

$$m = \frac{f(4)-f(-1)}{4-(-1)} = \frac{12-7}{5} = \frac{5}{5} = \boxed{1}$$

b. Write the equation of the secant line from $x = -1$ to $x = 4$.

use either $(4,12)$ or $(-1,7)$

$$\boxed{y-12 = (x-4) \quad \text{or} \quad y-7 = (x+1)}$$

c. Find the instantaneous rate of change for $f(x) = 2x^2 - 5x$ at $x = -1$

$$f(-1) = 7$$

$$\lim_{x \rightarrow -1} \frac{(2x^2 - 5x) - 7}{x + 1} = \lim_{x \rightarrow -1} \frac{2x^2 - 5x - 7}{x + 1} = \lim_{x \rightarrow -1} \frac{(x+1)(2x-7)}{(x+1)} = 2(-1) - 7 = \boxed{-9}$$

3. Given the function $f(x) = x^2 + 2x - 4$

a. Find the **slope** of $f(x)$ at $x = 3$ using the derivative definition at a point.

$$m = \lim_{x \rightarrow 3} \frac{(x^2 + 2x - 4) - (3^2 + 2(3) - 4)}{x - 3}$$

$$m = \lim_{x \rightarrow 3} \frac{x^2 + 2x - 4 - 11}{x - 3}$$

$$m = \lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x - 3}$$

$$\lim_{x \rightarrow 3} \frac{(x-3)(x+5)}{x-3} = 3+5 = \boxed{8}$$

$$f(3) = 3^2 + 2(3) - 4 = 9 + 6 - 4 = 11$$

b. Write the equation of the line **tangent** to the curve at $x = 3$.

$$m=8 \quad f(3)=11 \quad \boxed{y-11=8(x-3)}$$

c. Write the equation of the line **normal** to the curve at $x = 3$.

$$m_1 = -\frac{1}{8} \quad \boxed{y-11 = -\frac{1}{8}(x-3)}$$