Key

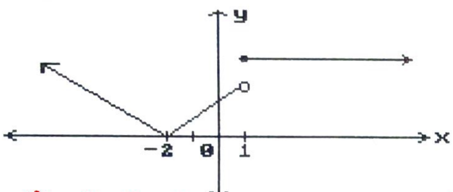
Drill_Continuity Calculus

State the 3 Conditions for $f(x)$ so that it is continuous at $x = a$. (Definition of continuity at a point.)

1. $f(a)$ exists
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

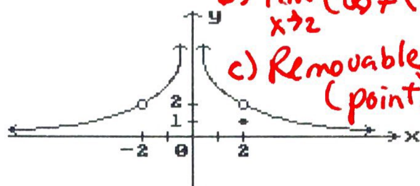
Problems #1 – 5. For each function:
a. State the location where the function is discontinuous.
b. State the reason why the function is discontinuous using the definition of continuity stated above at those locations.
c. State the type of discontinuity.

1.



a) at $x = 1$
b) $\lim_{x \to 1} f(x) = DNE$
c) Jump

2.

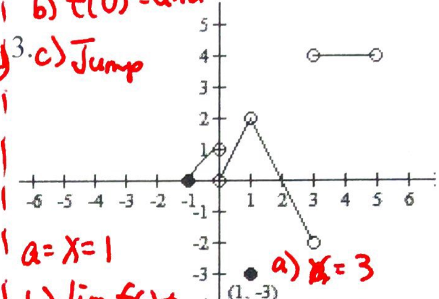


a) $x = 2$
b) $\lim_{x \to 2} f(x) \neq f(2)$
c) Removable (point)

a) $x = -2$
b) $f(-2) = und$
c) Removable (point)

a) $x = 0$
b) $\lim_{x \to 0} f(x) = \infty$ (DNE)
c) Infinite

3.



a) $x = 0$
b) $f(0) = und$
c) Jump

a $= x = 1$
b) $\lim_{x \to 1} f(x) \neq f(1)$
c) Removable (point)

a) $x = 3$
b) $f(3) = und$
c) Jump

4. $f(x) = \begin{cases} \dfrac{x^2 - 4}{x + 2}, & x \neq -2 \\ 3, & x = -2 \end{cases}$

$\lim_{x \to -2} \dfrac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} \stackrel{?}{=}$

$-2 - 2 = -4 \neq 3$

b) $\lim_{x \to -2} f(x) \neq f(-2)$

$\therefore$ a) Disc at $x = -2$

c) (Point)

5. $f(x) = \begin{cases} 0, & x < 0 \\ x^2 - 4x, & 0 \le x \le 4 \\ 4, & x > 4 \end{cases}$

$\lim_{x \to 0^-} 0 = 0$ $\quad \lim_{x \to 0^+} x^2 - 4x = 0$

$\lim_{x \to 4^-} x^2 - 4x:$ $16 - 16 = 0$

$\lim_{x \to 4^+} 4 = 4$) $\neq$

a) $x = 4$
b) $\lim_{x \to 4} f(x) = DNE$
c) Jump

6. $f(x) = \dfrac{x^2 - 7x + 6}{2x^2 - 12x}$

7. Find the value for ***a*** that would make the function continuous.

$f(x) = \begin{cases} x^2 + x + a, & x < 4 \\ x^3, & x \ge 4 \end{cases}$

$\lim_{x \to 4^-} x^2 + x + a = \lim_{x \to 4^+} x^3$

$4^2 + 4 + a = 4^3$

$20 + a = 64$

$-20 \qquad -20$

$\boxed{a = 44}$

8. Find the value for *k* that would make the function continuous.

$f(x) = \begin{cases} 6x + 8, \text{ if } x < -10 \\ kx + 6, \text{ if } x \ge -10 \end{cases}$

$\lim_{x \to -10^-} 6x + 8 = \lim_{x \to -10^+} kx + 6$

$6(-10) + 8 = -10k + 6$

$-52 = -10k + 6$

$-58 = -10k$

$k = \frac{58}{10} = \boxed{\frac{29}{5}}$