DAY: 2

LIMIT LAWS

Let a and c be real numbers and let f and g be functions with the following limits:

$$\lim_{x\to a} f(x) = L \quad \text{and} \quad \lim_{x\to a} g(x) = M$$

Then

LAW NO.	LIMIT OF A . . .	MATH	WORDS
1	Sum	$\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = L + M$	The limit of a sum is the sum of the limits.
2	Difference	$\lim_{x\to a}[f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = L - M$	The limit of a difference is the difference of the limits.
3	Constant (Scalar) Multiple	$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x) = cL$	The limit of a constant times a function is the constant times the limit of the function.
4	Product	$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = LM$	The limit of a product is the product of the limits.
5	Quotient	$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}$ if $M \neq 0$	The limit of a quotient is the quotient of the limits (provided the limit of the denominator is not equal to 0).

Let a be a real number and n be a positive integer and let f be a function with the following limit:

$$\lim_{x\to a} f(x) = L$$

Then

LAW NO.	LIMIT OF A . . .	MATH	WORDS
6	Power	$\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n = L^n$	The limit of a power is the power of the limit.
7	Root	$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{L}$ Note: If n is even, we assume $L > 0$.	The limit of a root is the root of the limit.

SPECIAL LIMITS

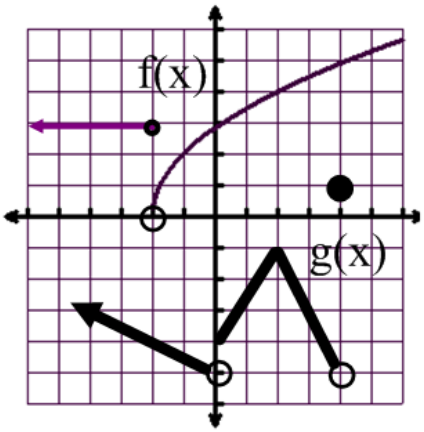
Let a and c be real numbers and n be a positive integer. Then

SPECIAL LIMIT NO.	WORDS	MATH
1	The limit of a constant function	$\lim_{x\to a} c = c$
2	The limit of the identity function	$\lim_{x\to a} x = a$
3	The limit of a power function	$\lim_{x\to a} x^n = a^n$
4	The limit of a radical function	$\lim_{x\to a}\sqrt[n]{x} = \sqrt[n]{a}$, where $a > 0$

DIRECT SUBSTITUTION: LIMITS OF POLYNOMIAL AND RATIONAL FUNCTIONS

FUNCTION	DIRECT SUBSTITUTION	RESTRICTIONS ON *a*
Polynomial: $f(x)$	$\lim_{x\to a} f(x) = f(a)$	a is any real number.
Rational: $f(x) = \frac{n(x)}{d(x)}$	$\lim_{x\to a} f(x) = f(a) = \frac{n(a)}{d(a)}$	a is any real number such that $d(a) \neq 0$.

Given the graphs of the functions, use the limit Laws to find the following limits, if they exist.



1. $\lim_{x\to 2}[f(x) - g(x)]$

2. $\lim_{x\to 4}\sqrt{f(x) + g(x)}$

3. $\lim_{x\to 0}[f(x)]^2$

4. $\lim_{x\to -3}\frac{f(x)}{g(x)}$

5. $\lim_{x\to -1}[g(x)f(x)]$

Find the limit <u>Algebraically</u> if it exists.

1. $\lim_{x\to -3} 2x^2 + x - 1$

2. $\lim_{x\to 7}\sqrt{x+2}$

3. $\lim_{x\to 1}\frac{x^2+x-2}{x-1}$

4. $\lim_{x\to 2}\frac{2x^2-5x+2}{5x^2-7x-6}$

5. $\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$

6. $\lim_{x\to 0}\frac{\sqrt{x^2+4}-2}{x^2}$

7. $\lim_{x\to 0}\frac{\frac{1}{x-1}+1}{x}$

#8 – 9 , find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

8. $f(x) = 5x + 2$

9. $f(x) = -x^2 + 2x + 3$

Evaluate the one-sided limits in order to find the limit, if it exists.

10. $\lim_{x\to 0} f(x), where\ f(x) = \begin{cases} -x+1, & x < 0 \\ x+1, & x > 0 \end{cases}$

11. $\lim_{x\to -4}\frac{|x+4|}{x+4}$

12. $\lim_{x\to \frac{\pi}{2}} f(x), where\ f(x) = \begin{cases} \sin x, & x < \frac{\pi}{2} \\ \cos x, & x > \frac{\pi}{2} \end{cases}$