AP CALCULUS AB - AP REVIEW 6

Work these on notebook paper. **No calculator** except for problems 68, 73, and 75.

65.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ $\left(\text{ft}/\text{sec}^2\right)$	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

(a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)|\, dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)|\, dt$ using a trapezoidal approximation with the three subintervals determined by the table.

(b) Using appropriate units, explain the meaning of $\int_{0}^{30} a(t)\, dt$ in terms of the car's motion. Find the exact value of $\int_{0}^{30} a(t)\, dt$.

(c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.

(d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

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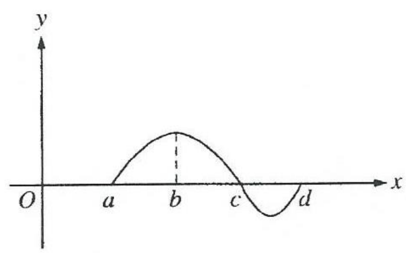
66. The graph of f is shown in the figure on the right.

If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a (B) b (C) c (D) d

(E) It cannot be determined from the information given.



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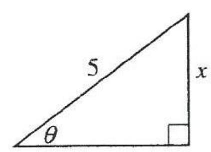
67. In the triangle shown on the right, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when $x = 3$ units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12



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68. (Calc) The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec (B) 26.447 ft/sec (C) 32.809 ft/sec (D) 40.671 ft/sec (E) 79.342 ft/sec

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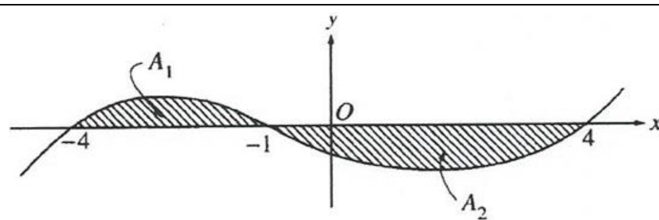
69.



The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2, $\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$

(A) A_1 (B) $A_1 - A_2$ (C) $2A_1 - A_2$ (D) $A_1 + A_2$ (E) $A_1 + 2A_2$

70. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

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71. $\lim\limits_{h\to 0} \frac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x)=\ln x$

(B) $f'(e)$, where $f(x)=\frac{\ln x}{x}$

(C) $f'(1)$, where $f(x)=\ln x$

(D) $f'(1)$, where $f(x)=\ln(x+e)$

(E) $f'(0)$, where $f(x)=\ln x$

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72. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

(A) $f(0)=0$

(B) $f'(c)=\frac{4}{9}$ for at least one c between -3 and 6

(C) $-1\le f(x)\le 3$ for all x between -3 and 6

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3

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73. (Calc) Let g be the function given by $g(x)=\int_0^x \sin(t^2)\,dt$ for $-1\le x\le 3$. On which of the following intervals is g decreasing?

(A) $-1\le x\le 0$

(B) $0\le x\le 1.772$

(C) $1.253\le x\le 2.171$

(D) $1.772\le x\le 2.507$

(E) $2.802\le x\le 3$

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74. If $f''(x)=x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) – 1 only

(B) 2 only

(C) – 1 and 0 only

(D) – 1 and 2 only

(E) – 1, 0, and 2 only

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75. (Calc) A particle moves along the x-axis so that any time $t>0$, its acceleration is given by $a(t)=\ln(1+2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is

(A) 0.462

(B) 1.609

(C) 2.555

(D) 2.886

(E) 3.346