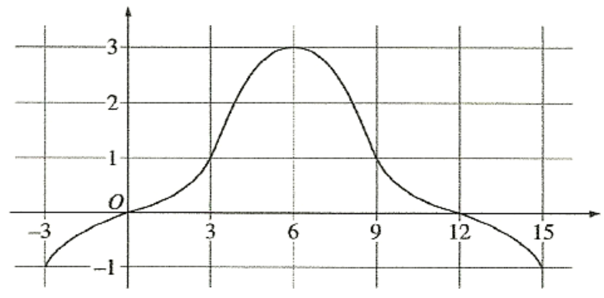
AP CALCULUS AB - AP REVIEW 5

Work the following on notebook paper, showing all work. Use your calculator only on problem 62.

54. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown on the right. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

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55. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

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56. $\int_1^e \left(\frac{x^2 - 1}{x}\right) dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $e^2 - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

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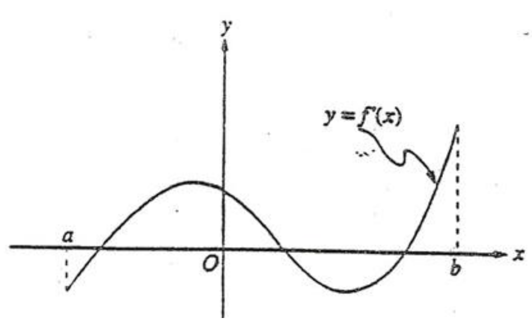
57. Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \le 0 \\ x & \text{for } x > 0 \end{cases}$. Which of the following statements about f is true?

(A) f is an odd function. (B) f is discontinuous at $x = 0$. (C) f has a relative maximum.

(D) $f'(0) = 0$ (E) $f'(x) > 0$ for $x \ne 0$.

58.



The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima

(B) Two relative maxima and one relative minimum

(C) Three relative maxima and one relative minimum

(D) one relative maximum and three relative minima

(E) Three relative maxima and two relative minima

59. (Calc) A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right) \text{ gallons per hour. During}$$

the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?
(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?
(c) At what time t, for $0 \le t \le 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.
(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

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60. The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$ (B) $V(t) = k\sqrt{V}$ (C) $\frac{dV}{dt} = k\sqrt{t}$
(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$ (E) $\frac{dV}{dt} = k\sqrt{V}$

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61. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

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62. (Calc) A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

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63. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

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64. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute (B) $-\frac{7}{24}$ feet per minute (C) $\frac{7}{24}$ feet per minute
(D) $\frac{7}{8}$ feet per minute (E) $\frac{21}{25}$ feet per minute