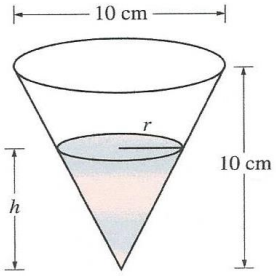
AP CALCULUS AB – AP REVIEW 4

Work the following on notebook paper, showing all work. **No calculator**.

41. A container has the shape of an open right circular cone, as shown in the figure on the right. The height of the container is 10 cm, and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}\frac{\text{cm}}{\text{hr}}$. (The volume of a cone of height h and radius r is given by $V=\frac{1}{3}\pi r^2 h$.)

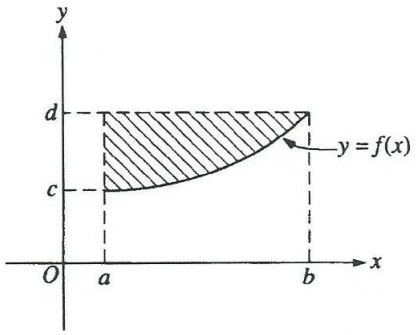


(a) Find the volume V of water in the container when h = 5 cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when h = 5 cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

42.



Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)\,dy$ (B) $\int_a^b \big(d-f(x)\big)\,dx$ (C) $f'(b)-f'(a)$

(D) $(b-a)\big[f(b)-f(a)\big]$ (E) $(d-c)\big[f(b)-f(a)\big]$

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43. If $x^3+3xy+2y^3=17$, then in terms of x and y, $\frac{dy}{dx}=$

(A) $-\frac{x^2+y}{x+2y^2}$ (B) $-\frac{x^2+y}{x+y^2}$ (C) $-\frac{x^2+y}{x+2y}$ (D) $-\frac{x^2+y}{2y^2}$ (E) $\frac{-x^2}{1+2y^2}$

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44. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx=$

(A) $2\sqrt{x^3+1}+C$ (B) $\frac{3}{2}\sqrt{x^3+1}+C$ (C) $\sqrt{x^3+1}+C$ (D) $\ln\sqrt{x^3+1}+C$ (E) $\ln\left(x^3+1\right)+C$

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45. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

46. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

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47. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) π (E) $\frac{3\pi}{2}$

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48. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

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49. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$ (B) $3t^2 - 2t + 4$ (C) $t^3 - t^2 + 4t + 6$
(D) $t^3 - t^2 + 9t - 20$ (E) $36t^3 - 4t^2 - 77t + 55$

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50. $\frac{d}{dx}\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\frac{1}{2\pi}\sin x$ (C) $\frac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

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51. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

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52. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

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53. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x