AP CALCULUS AB – AP REVIEW 3

Work the following on **notebook** **paper**, showing all work. Use your calculator only on problems 27 and 34.

27. (Calc) The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$. A pumping station adds sand to the beach at a rate modeled by the function S, given by $S(t) = \frac{15t}{1+3t}$. Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answer.

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28. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $-\frac{2x}{1+3y^2}$ (D) $-\frac{2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

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29. $\int_1^2 \frac{x^2-1}{x+1}\,dx =$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) ln 3

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30. If $\lim_{x\to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists (B) $f(x)$ is continuous at $x = a$. (C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$ (E) None of the above

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31. $\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2} - 5$ (C) $\sqrt{1+x^2}$ (D) $\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$ (E) $\frac{1}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{5}}$

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32. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval [0, 2] is

(A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26

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33. If $y = x^2 e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x\left(x+2e^x\right)$ (C) $xe^x(x+2)$ (D) $2x+e^x$ (E) $2x+e$

34. (Calc) A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piece-wise defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \le t \le 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.
(b) Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.
(c) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.
(d) Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 9000 liters.

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35. If $y = \dfrac{\ln x}{x}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{x^2}$ (C) $\dfrac{\ln x - 1}{x^2}$ (D) $\dfrac{1 - \ln x}{x^2}$ (E) $\dfrac{1 + \ln x}{x^2}$

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36. $\displaystyle\int \frac{x}{\sqrt{3x^2+5}}\,dx =$

(A) $\dfrac{1}{9}(3x^2+5)^{3/2} + C$ (B) $\dfrac{1}{4}(3x^2+5)^{3/2} + C$ (C) $\dfrac{1}{12}(3x^2+5)^{1/2} + C$

(D) $\dfrac{1}{3}(3x^2+5)^{1/2} + C$ (E) $\dfrac{3}{2}(3x^2+5)^{1/2} + C$

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37. $\displaystyle\lim_{h \to 0} \frac{\tan 3(x+h) - \tan(3x)}{h} =$

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

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38. What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) -6 (B) -2 (C) $\dfrac{3}{2}$ (D) $\dfrac{9}{4}$ (E) $\dfrac{9}{2}$

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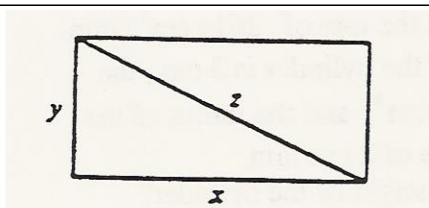
39. If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$, then $\int_1^{3} f(x)\,dx =$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11

40.



The sides of the rectangle above increase in such a way that $\dfrac{dz}{dt} = 1$ and $\dfrac{dx}{dt} = 3\dfrac{dy}{dt}$.

At the instant when $x = 4$ and $y = 3$, what is the value of $\dfrac{dx}{dt}$?

(A) $\dfrac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5