

Section 3.4

Answers to Evens on Assignment #3

$$\begin{aligned} 8. \quad Q(t) &= 200(30-t)^2 \\ &= 200(900 - 60t + t^2) \\ &= 180,000 - 12,000t + 200t^2 \end{aligned}$$

$$Q'(t) = -12,000 + 400t$$

The rate of change of the amount of water in the tank after 10 minutes is $Q'(10) = -8000$ gallons per minute.

Note that $Q'(10) < 0$, so the rate at which the water is running *out* is positive. The water is running out at the rate of 8000 gallons per minute.

The average rate for the first 10 minutes is

$$\frac{Q(10) - Q(0)}{10 - 0} = \frac{80,000 - 180,000}{10} = -10,000 \text{ gal/min.}$$

The water is flowing out at an average rate of 10,000 gallons per minute over the first 10 min.

16. Moon:

$$\begin{aligned} s(t) &= 0 \\ 832t - 2.6t^2 &= 0 \\ 2.6t(320 - t) &= 0 \\ t = 0 \text{ or } t = 320 \end{aligned}$$

It takes 320 seconds to return.

Earth:

$$\begin{aligned} s(t) &= 0 \\ 832t - 16t^2 &= 0 \\ 16t(52 - t) &= 0 \\ t = 0 \text{ or } t = 52 \end{aligned}$$

It takes 52 seconds to return.

18. (a) 190 ft/sec

(b) 2 seconds

(c) After 8 seconds, and its velocity was 0 ft/sec then

(d) After about 11 seconds, and it was falling 90 ft/sec then

(e) About 3 seconds (from the rocket's highest point)

(f) The acceleration was greatest just before the engine stopped. The acceleration was constant from $t = 2$ to $t = 11$, while the rocket was in free fall.

Calculus

$$20. \quad \text{(a)} \quad v(t) = \frac{ds}{dt} = \frac{d}{dt}(-t^3 + 7t^2 - 14t + 8)$$

$$v(t) = -3t^2 + 14t - 14$$

$$\text{(b)} \quad a(t) = \frac{dv}{dt} = \frac{d}{dt}(-3t^2 + 14t - 14)$$

$$a(t) = -6t + 14$$

$$\text{(c)} \quad v(t) = -3t^2 + 14t - 14 = 0$$

$$t \approx 1.451, 3.215$$

(d) The particle starts at the point $s = 8$ when $t = 0$ and moves left until it stops at $s = -0.631$ when $t = 1.451$, then it moves right to the point $s = 2.113$ when $t = 3.215$ where it stops again, and finally continues left from there on.

$$24. \quad a(t) = v'(t) = 6t^2 - 18t + 12$$

Find when acceleration is zero.

$$6t^2 - 18t + 12 = 0$$

$$6(t^2 - 3t + 2) = 0$$

$$6(t-1)(t-2) = 0$$

$$t = 1 \text{ or } t = 2$$

At $t = 1$, the speed is $|v(1)| = |0| = 0$ m/sec.

At $t = 2$, the speed is $|v(2)| = |-1| = 1$ m/sec.

$$34. \quad \text{(a)} \quad \frac{dV}{dr} = \frac{d}{dr} \left(\frac{4}{3} \pi r^3 \right) = 4\pi r^2$$

When $r = 2$, $\frac{dV}{dr} = 4\pi(2)^2 = 16\pi$ cubic feet of volume per foot of radius.

(b) The increase in the volume is

$$\frac{4}{3} \pi (2.2)^3 - \frac{4}{3} \pi (2)^3 \approx 11.092 \text{ cubic feet.}$$