Analyzing f, f', and f'' Solutions

We have intentionally included more material than can be covered in most Student Study Sessions to account for groups that are able to answer the questions at a faster rate. Use your own judgment, based on the group of students, to determine the order and selection of questions to work in the session. Be sure to include a variety of types of questions (multiple choice, free response, calculator, and non-calculator) in the time allotted.

Multiple Choice

1. D (1993 BC9 appropriate for AB)

   Since $f'(x) = \frac{2}{3}x^{\frac{-1}{3}} = \frac{2}{3\sqrt[3]{x}}$, $f'(0)$ does not exist.

2. B (1973 AB22)

   Determine the interval(s) where $f''(x) > 0$.

   $f'(x) = 15x^4 - 60x^2$

   $f''(x) = 60x^3 - 120x$

   $60x(x^2 - 2) = 0$

   $x = 0$ or $x = \pm\sqrt{2}$

   $f''(x) = 60x^3 - 120x > 0$ on the intervals $\left(-\sqrt{2}, 0\right) \cup \left(\sqrt{2}, \infty\right)$.

3. D (1997 AB22)

   Determine the interval(s) where $f'(x) > 0$.

   $f'(x) = (x^2 - 3)(-e^{-x}) + (2x)(e^{-x})$

   $f'(x) = -e^{-x}(x^2 - 2x - 3)$

   $0 = -e^{-x}(x-3)(x+1)$

   $x = 3$ and $x = -1$

   $f'(x) > 0$ on $(-1, 3)$

4. C (1998 AB19)

   A point of inflection occurs when $f''(x)$ changes sign. In this case, $f''(x)$ has a double root at $x = 2$ and changes sign only at $x = -1$ and $x = 0$.

5. C (1998 AB22)

   The function, $f(x)$, is increasing when $f'(x) > 0$.

   $f'(x) = 4x^3 + 2x$

   $2x(2x^2 + 1) = 0$

   $x = 0$

   $f'(x) = 4x^3 + 2x$ is positive on the interval $(0, \infty)$.

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6. E (1988 AB4)
   The function, y, is concave downward when y'' is negative.
   $y'' = \dfrac{-10}{(x-2)^3}$ is negative for $x > 2$.

7. B (1993 BC22)
   The function f is decreasing when f' is negative.
   $f'(x) = x^2e^x + 2xe^x$
   $xe^x(x+2) = 0$
   $x = 0$ or $x = -2$
   Thus $f'(x) < 0$ on the interval $(-2, 0)$.

8. B (1997 BC80 appropriate for AB)
   Using the derivative function on the calculator, graph f'. Calculate the smallest x value where f' changes from increasing to decreasing or decreasing to increasing.
   or
   Using the derivative function on the calculator, graph $f''(x)$. Calculate the smallest x value where $f''(x)$ changes sign.

9. E (2003 BC83 appropriate for AB)
   Rolle's Theorem guarantees an extreme value on the interval $(0, 4)$ given that f is continuous and differentiable on the closed interval $[0, 4]$ and $f(0) = f(4) = 2$.

10. C (2003 BC86 appropriate for AB)
    Analyze the graph of $f'(x)$ to determine the number of times $f'(x)$ changes from increasing to decreasing or decreasing to increasing on the interval $(-1.8, 1.8)$
    or
    Use the derivative function on the calculator to graph $f''(x)$ to determine the number of times $f''(x)$ changes signs on the interval $(-1.8, 1.8)$

11. C (1997 AB85)
    $f'(x)$ changes sign from positive to negative only at $x = 0.91$.

12. B (1998 AB89)
    The graph of $y = x^2 - 4$ is a parabola that changes from positive to negative at $x = -2$ and from negative to positive at $x = 2$. Since g is always negative, f' changes sign opposite to the way $y = x^2 - 4$ does. Thus, f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.

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Free Response

13. 1975 AB4/BC1

(a) $y' = 1 + \cos x$

$0 = 1 + \cos x$

$-1 = \cos x$

$\pi = x$

At $x = \pi$, there is not an extreme value since y' does not change signs there.

$y\left(-\frac{\pi}{2}\right) = -\frac{\pi}{2} - 1 \qquad y\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} - 1$

Absolute minimum value at $\left(-\frac{\pi}{2}, -\frac{\pi}{2} - 1\right)$

Absolute maximum value at $\left(\frac{3\pi}{2}, \frac{3\pi}{2} - 1\right)$

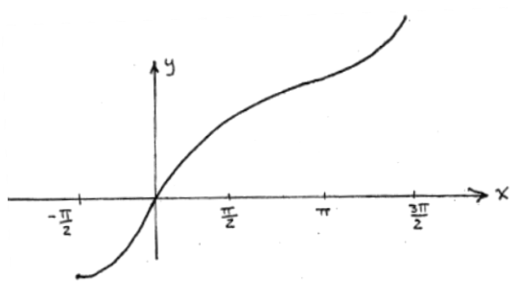
(b) $y'' = -\sin x$

$-\sin x = 0$

$x = 0, \pi$

y has points of inflection at $(0, 0)$ and (π, π) since y'' changes signs at $x = 0$ and $x = \pi$ on the interval from $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

(c)



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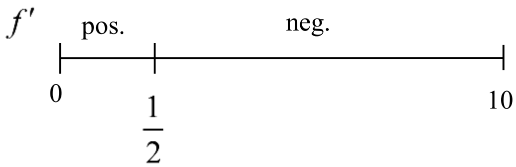
14. 1979 AB2

(a) $f(x) = xe^{-2x}$

$f'(x) = e^{-2x} - 2xe^{-2x}$

$e^{-2x}(1-2x) = 0$

$e^{-2x} \neq 0$ and $x = \frac{1}{2}$



f is increasing on $\left(0, \frac{1}{2}\right)$ since $f'(x) > 0$ in this interval and f is decreasing on $\left(\frac{1}{2}, 10\right)$ since $f'(x) < 0$ on this interval.

(b) $f\left(\frac{1}{2}\right) = \frac{1}{2e}$ is a relative maximum from part (a)

$f(0) = 0$

$f(10) = \frac{10}{e^{20}}$

Therefore, $\left(\frac{1}{2}, \frac{1}{2e}\right)$ is the absolute maximum and $(0, 0)$ is the absolute minimum.

No point values available.

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15. 1999 AB4

(a) Slope at $x=0$ is $f'(0)=-3$ At $x=0$, $y=2$ $y-2=-3(x-0)$	1: equation
(b) No. Whether $f''(x)$ changes sign at $x=0$ is not known. The only given values of $f''(x)$ is $f''(0)=0$.	2 { 1: answer 1: explanation
(c) $g'(x)=e^{-2x}(3f(x)+2f'(x))$ $g'(0)=e^{0}(3f(0)+2f'(0))$ $3(2)+2(-3)=0$ $y-4=0(x-0)$ $y=4$	2 { 1: $g'(0)$ 1: equation
(d) $g'(x)=e^{-2x}(3f(x)+2f'(x))$ $g''(x)=(-2e^{-2x})(3f(x)+2f'(x))$ $+e^{-2x}(3f'(x)+2f''(x))$ $=e^{-2x}(-6f(x)-f'(x)+2f''(x))$ $g''(0)=e^{0}[(-6)(2)-(-3)+2(0)]=-9$ Since $g'(0)=0$ and $g''<0$, g does have a local maximum at $x=0$	4 { 2: verify derivative 0/2 product or chain rule error < -1> algebra error 1: $g'(0)=0$ and $g''(0)$ 1: Answer and reasoning

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16. 2001 AB4/BC4

(a) $h'(x) = 0$ at $x = \pm\sqrt{2}$

$h'(x)$	$-$	0	$+$	und	$-$	0	$+$
x		$-\sqrt{2}$		0		$\sqrt{2}$	

Local minima at $x = \sqrt{2}$ and at $x = -\sqrt{2}$

4 { 1: $x = \pm\sqrt{2}$
1 analysis
1: conclusions
< -1> not dealing with discontinuity at 0 }

(b) $h''(x) = 1 + \dfrac{2}{x^2} > 0$ for all $x \neq 0$.

Therefore, the graph of h is concave up for all $x \neq 0$.

3 { 1: $h''(x)$
1: $h''(x) > 0$
1: answer }

(c) $h'(4) = \dfrac{16-2}{4} = \dfrac{7}{2}$

$y + 3 = \dfrac{7}{2}(x - 4)$

1: tangent line equation

(d) The tangent line is below the graph because the graph of h is concave up for $x > 4$.

1: answer with reason

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