Accumulation Solutions

We have intentionally included more material than can be covered in most Student Study Sessions to account for groups that are able to answer the questions at a faster rate. Use your own judgment, based on the group of students, to determine the order and selection of questions to work in the session. Be sure to include a variety of types of questions (multiple choice, free response, calculator, and non-calculator) in the time allotted.

Multiple Choice Answers

1. D 1998 AB9/BC9

   Using dimensional analysis, $\left(\frac{\text{barrels}}{\text{hour}}\right)\text{hours} = \text{barrels}$. The amount can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares, each with an area of 600 barrels, so the total is about 3000 barrels.

2. C 2008 AB10

   Since $f(x)$ is decreasing, the Right Riemann Sum $< \int_1^3 f(x)\,dx <$ Left Riemann Sum.

   Since $f(x)$ is decreasing and concave down, the Right Riemann Sum $<$ both the Midpoint Riemann Sum and the Trapezoidal Sum.

3. E 1988 BC18 (appropriate for AB)

   $\frac{1}{2}(2)\left(\frac{e^4}{2}+2\frac{e^2}{2}+2\frac{e^0}{2}+\frac{e^{-2}}{2}\right)=\frac{1}{2}(e^4+2e^2+2e^0+e^{-2})$

4. C 1998 AB85/BC85

   $\frac{1}{2}(3)(10+30)+\frac{1}{2}(2)(30+40)+\frac{1}{2}(1)(40+20)=160$

5. D 1993 AB36

   Rectangle approximation = $e^0+e^1=1+e$

   Trapezoid approximation = $\frac{(1+2e+e^4)}{2}$

   Difference = $\frac{e^4-1}{2}=26.799$

6. B 2003 BC90 (appropriate for AB)

   I. False. The area accumulated from 0 to 1 is greater than the area accumulated from 0 to 0.
   II. True. The area accumulated from 0 to 2 is greater than the area accumulated from 0 to 1.
   III. False. The area accumulated from 0 to 3 is greater than the area accumulated from 0 to 1.

7. A 2003 AB85/BC85

Since using trapezoids give an over-approximation, the graph is concave up. Since right Riemann sums give an under-approximation, the graph is decreasing. The only graph that is concave up and decreasing is choice (A).

8. E 1998 BC91 (appropriate for AB)

$$11\frac{\text{ft}}{\text{sec}} + 2\text{ sec}\left(5\frac{\text{ft}}{\text{sec}} + 2\frac{\text{ft}}{\text{sec}} + 8\frac{\text{ft}}{\text{sec}}\right) = 41\frac{\text{ft}}{\text{sec}}$$

Free Response Solutions

9. 2007 AB5/BC5

(c) $\int_0^{12} r'(t)dt \approx$ $2(4.0)+3(2.0)+2(1.2)+4(0.6)+1(0.5)=19.3\text{ft}$ $\int_0^{12} r'(t)dt$ is the change in the radius, in feet, from $t=0$ to $t=12$ minutes.	2 { 1: approximation 1: explanation
(d) Since r is concave down, r' is decreasing on $0<t<12$. Therefore, this approximation, 19.3 ft, is less then $\int_0^{12} r'(t)dt$.	1: conclusion with reason
Unit of ft^3/min in part (b) or ft in part (c)	1: units in (b) or (c)

10. 2011B AB5/BC5

(b) $\int_0^{60} \lvert v(t)\rvert\, dt$ is the total distance, in meters, that Ben rides over the 60-second interval $t=0$ to $t=60$. $\int_0^{60} \lvert v(t)\rvert\, dt \approx 2.0\cdot 10+2.3(40-10)+2.5(60-40)=139$ meters	2 { 1: meaning of integral 1: approximation
	1: units in (a) or (b)

11. 2010 AB2

(b) $\frac{1}{8}\int_0^8 E(t)\,dt \approx$ $\frac{1}{8}\left(2\cdot\frac{E(0)+E(2)}{2}+3\cdot\frac{E(2)+E(5)}{2}+2\cdot\frac{E(5)+E(7)}{2}+1\cdot\frac{E(7)+E(8)}{2}\right)$ $=10.687$ or 10.688 $\frac{1}{8}\int_0^8 E(t)\,dt$ is the average number of hundreds of entries in the box between noon and 8 p.m.	$3\begin{cases}1: \text{ trapezoidal sum}\\1: \text{ approximation}\\1: \text{ meaning}\end{cases}$
(c) $23-\int_8^{12} P(t)\,dt = 23-16 = 7$ hundred entries	$2\begin{cases}1: \text{ integral}\\1: \text{ answer}\end{cases}$

12. 2006 AB4/BC4

(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t=10$ seconds to $t=70$ seconds. A midpoint Riemann sum is $20\left[v(20)+v(40)+v(60)\right]$ $=20[22+35+44]=2020$ ft	$3\begin{cases}1: \text{ explanation}\\1: \text{ uses } v(20),\ v(40),\ v(60)\\1: \text{ value}\end{cases}$
(c) Let $v_B(t)$ be the velocity of rocket B at time t. $v_B(t)=\int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1}+C$ $2=v_B(0)=6+C$ $v_B(t)=6\sqrt{t+1}-4$ $v_B(80)=50>49=v(80)$ Rocket B is traveling faster at time $t=80$ seconds.	$4\begin{cases}1: \ 6\sqrt{t+1}\\1: \text{ constant of integration}\\1: \text{ uses initial condition}\\1: \text{ finds } v_B(80)\text{, compares to } v(80)\text{, and draws a conclusion}\end{cases}$
Units of ft/sec^2 in (a) and ft in (b)	1: units in (a) and (b)

13. 2010 AB3

(a) $\int_0^3 r(t)\,dt = 2\cdot\frac{1000+1200}{2}+\frac{1200+800}{2} = 3200$ people	$2\begin{cases}1: & \text{integral}\\ 1: & \text{answer}\end{cases}$
(b) The number of people waiting in line is increasing because people move onto the ride at a rate of 800 people per hour and for $2<t<3$, $r(t)>800$.	1: answer with reason
(c) $r(t)=800$ only at $t=3$ For $0\le t<3$, $r(t)>800$. For $3<t\le 8$, $r(t)<800$. Therefore, the line is longest at time $t=3$. There are $700+3200-800\cdot 3=1500$ people waiting in line at time $t=3$.	$3\begin{cases}1: & \text{identifies } t=3\\ 1: & \text{number of people in line}\\ 1: & \text{justification}\end{cases}$
(d) $0=700+\int_0^t r(s)\,ds-800t$	$3\begin{cases}1: & 800t\\ 1: & \text{integral}\\ 1: & \text{answer}\end{cases}$