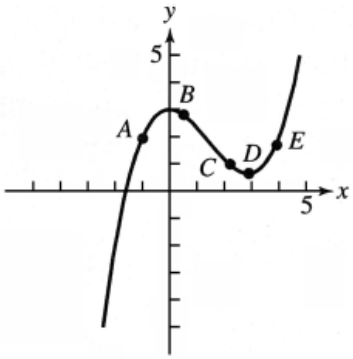
Advanced Placement Calculus AB Test

Section I–Part A (55 minutes)

Choose the best answer for each question. Your score is determined by subtracting one-fourth of the number of wrong answers from the number of correct answers. **Calculators are not permitted.**

1.



For the graph shown, at which point is it true that $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$?

(A) A **(B)** B **(C)** C **(D)** D **(E)** E

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2. Find the area of the region bounded by the x-axis and the graph of $y = (x+1)(x-2)^2$.

(A) $\frac{5}{4}$ **(B)** $2\frac{3}{4}$ **(C)** $5\frac{1}{4}$ **(D)** $6\frac{1}{4}$ **(E)** $6\frac{3}{4}$

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3. Which of the following is an antiderivative of $x^2 \sec^2 x^3$?

(A) $2x\sec^2 x^3 + 6x^4 \sec^2 x^3 \tan x^3$

(B) $2x\sec^2 x^3 + 6x^3 \sec x^3$

(C) $\frac{1}{3}\tan x^3 - 5$

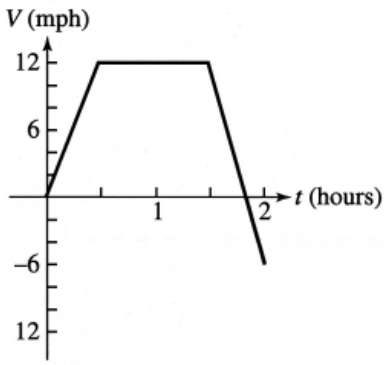
(D) $3\tan x^3 + \pi$

(E) $-\frac{1}{3}\cot x^3 + 4$

4. Line L is tangent to the curve defined by $2xy^2 - 3y = 18$ at the point (3, 2). The slope of line L is

(A) $\frac{21}{8}$ **(B)** $\frac{32}{3}$ **(C)** $-\frac{10}{21}$ **(D)** $\frac{8}{21}$ **(E)** $-\frac{8}{21}$

5.



A bicyclist rides along a straight road starting from home at $t = 0$. The graph above shows the bicyclist's velocity as a function of t. How far from home is the bicyclist after 2 hours?

(A) 13 miles **(B)** 16.5 miles **(C)** 17.5 miles **(D)** 18 miles **(E)** 20 miles

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6. Find the value of x at which the graph of $y = \frac{1}{x} + \sqrt{x}$ has a point of inflection.

(A) 2 **(B)** $4^{2/3}$ **(C)** 4 **(D)** 6 **(E)** 8

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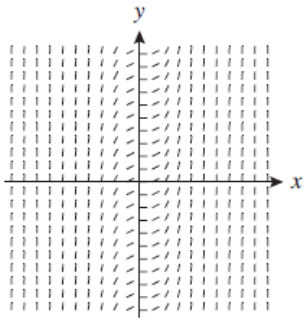
7. Find $\lim_{x \to \infty} \frac{2x - 4x^3}{8x^3 + 4x^2 - 3x}$.

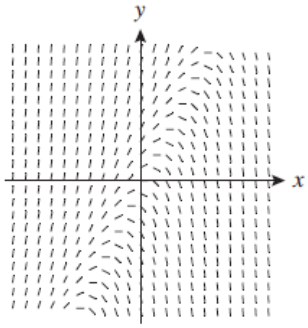
(A) $\frac{2}{3}$ **(B)** $\frac{3}{2}$ **(C)** 1 **(D)** $-\frac{1}{2}$ **(E)** $-\frac{3}{4}$

8. Which of the following is a slope field for the differential equation $\frac{dy}{dx} = -2x + y$?

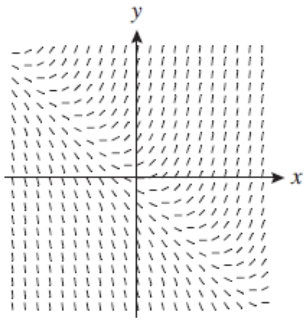
(A)



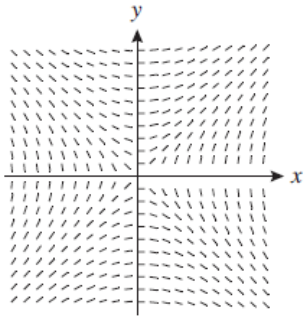
(B)



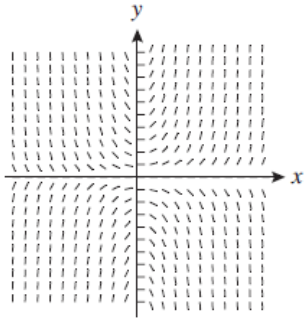
(C)



(D)



(E)



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9. Let $f(x) = \cos(3\pi x^2)$. Find $f'\left(\frac{1}{3}\right)$.

(A) $-\sqrt{3}\pi$ **(B)** $\sqrt{3}\pi$ **(C)** 0 **(D)** $-\frac{\sqrt{3}\pi}{2}$ **(E)** $-\pi$

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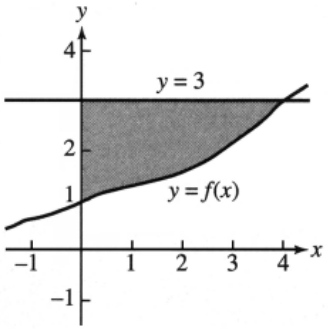
10. Assume that $f(x)$ is a one-to-one function. The area of the shaded region is equal to which of the following definite integrals?



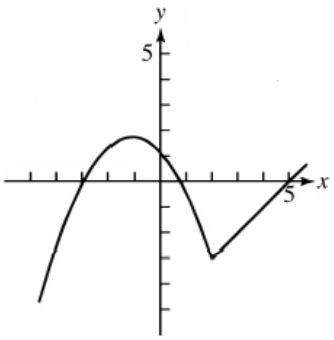
I. $\int_0^4 [f(x) - 3]\,dx$

II. $\int_4^0 [f(x) - 3]\,dx$

III. $\int_1^3 f^{-1}(y)\,dy$

(A) I only **(B)** II only **(C)** III only **(D)** I and III **(E)** II and III

11.



The graph of a function $y = f(x)$ is shown above. Which of the following are true for the function f?

I. $f'(2)$ is defined.

II. $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

III. $f'(x) < 0$ for all x in the open interval $(-1, 2)$.

(A) I only **(B)** II only **(C)** III only **(D)** II and III **(E)** I, II and III

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12. Let $f(x) = \sin^{-1} x$. Find $f'\left(\frac{\sqrt{2}}{2}\right)$.

(A) $\frac{\pi}{4}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{1}{2}$ **(D)** $\sqrt{2}$ **(E)** Undefined

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13. Evaluate $\int (\cos x - e^{2x})\,dx$.

(A) $-\sin x - \frac{1}{2}e^{2x} + C$

(B) $\sin x - \frac{1}{2}e^{2x} + C$

(C) $-\sin x - 2e^{2x} + C$

(D) $\sin x - 2e^{2x} + C$

(E) $-\cos x - \frac{1}{2}e^{2x} + C$

14. Let $f(x) = e^{x^3 - 2x^2 - 4x + 5}$. Then f has a local minimum at $x =$

(A) -2 **(B)** $-\frac{2}{3}$ **(C)** $\frac{2}{3}$ **(D)** 1 **(E)** 2

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15. The acceleration of a particle moving along the x-axis is $a(t) = 12t - 10$.
At $t = 0$, the velocity is 4.
At $t = 1$, the position is $x = 8$.
Find the position at $t = 2$.

(A) 5 **(B)** 4 **(C)** 10 **(D)** 11 **(E)** 7

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16. Let f be differentiable for all real numbers. Which of the following must be true for any real numbers a and b?

I. $\int_2^a f(x)\,dx = \int_2^b f(x)\,dx + \int_b^a f(x)\,dx$

II. $\int_a^b ([f(x)]^2 + f'(x))\,dx = [f(b)]^2 - [f(a)]^2$

III. $\int_a^b 3f(x)\,dx = 3\int_a^b f(x)\,dx$

(A) I only **(B)** II only **(C)** I and II **(D)** I and III **(E)** I, II, and III

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17. Find an equation of the line normal to the graph of $y = \frac{3x}{x^2 - 6}$ at $x = 3$.

(A) $5x + y = 18$ **(B)** $5x - y = 12$ **(C)** $5x + 3y = 24$ **(D)** $x - 5y = -12$ **(E)** $x + y = 6$

18. Let $g(x) = \lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$. For what value of x does $g(x) = 2$?

(A) $x = 1$ **(B)** $x = 2$ **(C)** $x = 3$ **(D)** $x = 4$ **(E)** $x = 5$

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19. Let f be a differentiable function of x that satisfies $f(1) = 7$ and $f(4) = 3$. Which of the following conditions would guarantee that the tangent line at $x = c$ is parallel to the secant line joining $(1, f(1))$ to $(4, f(4))$?

(A) $f(c) = \dfrac{3}{2}$ **(B)** $f(c) = 5$ **(C)** $f'(c) = -\dfrac{3}{4}$ **(D)** $f'(c) = -\dfrac{4}{3}$ **(E)** $f(c) = -\dfrac{4}{3}$

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20. Let $f(x) = x^3 - 12x$. Which statement about this function is false?

(A) The function has a relative minimum at $x = 2$.

(B) The function is increasing for values of x between -2 and 2.

(C) The function has two relative extrema.

(D) The function is concave upward for $x > 0$.

(E) The function has one inflection point.

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21. $\int_2^3 8x(x^2 - 5)\,dx =$

(A) $\dfrac{74}{3}$ **(B)** 30 **(C)** 90 **(D)** 112 **(E)** $\dfrac{370}{3}$

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22. Let $f(x) = \dfrac{d}{dx}\int_0^x \sqrt{t^2 + 16}\,dt$. What is $f(-3)$?

(A) -5 **(B)** -4 **(C)** 3 **(D)** 4 **(E)** 5

23. If $\frac{dy}{dx} = xy^2$ and $y = -\frac{1}{3}$ when $x = 2$, what is y when $x = 4$?

(A) $-\frac{1}{3}$ **(B)** $-\frac{1}{5}$ **(C)** $-\frac{1}{9}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{9}$

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24. Use the Trapezoidal Rule with $n = 3$ to approximate the area between the curve $y = x^2$ and the x-axis for $1 \le x \le 4$.

(A) 14 **(B)** 21 **(C)** 21.5 **(D)** 29 **(E)** 30

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25. Let $f(x)$ be a continuous function that is defined for all real numbers x.

If $f(x) = \frac{x^2 - x - 6}{x^2 - 5x + 6}$ when $x^2 - 5x + 6 \neq 0$, what is $f(3)$?

(A) 5 **(B)** 4 **(C)** 2 **(D)** 1 **(E)** 0

26. Find the derivative of $\cos^3 2x$.

(A) $-\sin^3 2x$

(B) $-6\cos^2 2x$

(C) $6\cos^2 2x \sin 2x$

(D) $-3\cos^2 2x \sin 2x$

(E) $-6\cos^2 2x \sin 2x$

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27. Let f be a twice-differentiable function whose derivative $f'(x)$ is increasing for all x. Which of the following must be true of all x?

I. $f(x) > 0$

II. $f'(x) > 0$

III. $f''(x) > 0$

(A) I only **(B)** II only **(C)** III only **(D)** I and II **(E)** II and III

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28. The function $f(x) = x^3 - 6x^2 + 9x - 4$ has a local maximum at

(A) $x = 0$ **(B)** $x = 1$ **(C)** $x = 2$ **(D)** $x = 3$ **(E)** $x = 4$