4.1 Arc Length-Angular Speed Math Analysis Name:
Notes/Examples

Arc Length

Definition: If a central angle $\boldsymbol{\theta}$ in a circle with radius $\boldsymbol{r}$ intercepts an arc on the circle of length $\boldsymbol{s}$, then the arc length $\boldsymbol{s}$ if given by

$$\boxed{\boldsymbol{s = r\theta\,,\ where\ \theta\ is\ given\ in\ radians}}$$

Ex: Find the exact arc length made by the indicated central angle and radius.

$\theta = 14°\,,\ \ r = 15\ in$

Area of Circular Sector

Definition: The area of a sector of a circle with radius $\boldsymbol{r}$ and central angle $\boldsymbol{\theta}$ is given by

$$\boxed{\boldsymbol{A = \frac{1}{2}r^2\theta\,,\ where\ \theta\ is\ given\ in\ radians}}$$

Ex. If a sprinkler head rotates 75° and has enough pressure to keep a constant 20-ft spray, what is the area of the sector of lawn it can water? Round to the nearest hundredth square foot.

Linear Speed

Definition: If a point ***P*** moves along the circumference of a circle at a constant speed, then the linear speed, $\boldsymbol{v}$ is given by

$$\boxed{\boldsymbol{v = \frac{s}{t}\,,\ where\ s = arc\ length\ and\ t = time}}$$

Ex: 1. A car travels at a constant speed around a circular track with circumference equal to 3 miles. If the car records a time of 12 minutes for 7 laps, what is the linear speed of the car in miles per hour?

Ex: 2. Find the distance traveled (arc length) of a point that moves with constant speed ***v*** along a circle in time ***t***. $v = 5.6\frac{ft}{s}$; $t = 2\ min$

Angular Speed

Definition: If a point ***P*** moves along the circumference of a circle at a constant speed, then the central angle $\boldsymbol{\theta}$ that is formed with the terminal side passing through point ***P*** also changes over some time ***t*** at a constant speed. The angular speed **ω** (omega) is given by:

$$\boxed{\boldsymbol{\omega = \frac{\theta}{t}\ ,\ where\ \theta\ is\ given\ in\ radians}}$$

Ex: Find the angular speed (radians/sec) associated with rotating a central $\boldsymbol{\theta}$ in time ***t***.
$\theta = 60°$; $t = 0.2\ sec$

Relationship Between Linear and Angular Speed

$$\boxed{\boldsymbol{v = r\omega \quad or \quad \omega = \frac{v}{r}\ ,\ only\ when\ \theta\ is\ given\ in\ radians}}$$

Ex: 1. Find the linear speed of a point traveling at a constant speed along the circumference of a circle with the given radius and angular speed. $\omega = \frac{3\pi\ rad}{4\ sec}$; $r = 8\ cm$

Ex: 2 A truck comes standard with tires that have a diameter of 25.7 inches (17” rims). If the owner decides to upgrade to tires with diameter of 28.2 inches (19” rims) without having the onboard computer updated, how fast will the truck actually be traveling when the speedometer reads 75 mph?