1_3GN_Infinite Limits and Asymptotes Calculus AB

I. Limits to $\pm\infty$: To find limits as x approaches $+\infty$ or $-\infty$ for rational functions:

- If the degree of the numerator is larger than the degree of the denominator, the limit will be $+\infty$ or $-\infty$. Test a point to determine whether the value is positive or negative.
- If the degree of the denominator is larger than the degree of the numerator, the limit will be 0.
- If the degrees of the numerator and the denominator are equal, the limit will be the ratio of the coefficients of the leading terms.

Examples:

1a. $\lim_{x \to \infty} \frac{x^2-4}{x^3+9}$ b. $\lim_{x \to -\infty} \frac{x^2-4}{x^3+9}$

2a. $\lim_{x \to \infty} \frac{3x^2-6x+2}{x^2}$ b. $\lim_{x \to -\infty} \frac{3x^2-6x+2}{x^2}$

3a. $\lim_{x \to \infty} \frac{x^3}{\sqrt{x^6+3}}$ b. $\lim_{x \to -\infty} \frac{x^3}{\sqrt{x^6+3}}$

4a. $\lim_{x \to \infty} \frac{x^5}{x^4-6x+9}$ b. $\lim_{x \to -\infty} \frac{x^5}{x^4-6x+9}$

II. Asymptotes: (for rational functions only)

A. <u>Vertical</u>

If the limit as x approaches a constant, c, is $+\infty$ or $-\infty$, then x = c is a vertical asymptote.
** If the denominator equals 0 at x = c **AND** there is no way to cancel the 0 from the denominator, then x = c is a vertical asymptote. **

1. $f(x) = \frac{1}{x+2}$ 2. $f(x) = \frac{x-2}{x^2-4}$

Vertical Asymptote(s): Vertical Asymptote(s):

B. Horizontal

If the limit as x approaches +∞ or -∞ is a constant, c, then **y = c** is a horizontal asymptote.
The degree of the numerator must be less than or equal to the degree of the denominator.
You must check for crossing with horizontal asymptotes.

1. $f(x) = \frac{x^2-4}{x^3+1}$

$\lim_{x \to \infty} \frac{x^2-4}{x^3+1} =$

Horizontal Asymptote(s):

2. $f(x) = \frac{4x^2+2}{x^2-9}$

$\lim_{x \to \infty} \frac{4x^2+2}{x^2-9} =$

Horizontal Asymptote(s):

*To check for cross point(s), set the function = to the Limit, then solve for x. The cross point will be $(x\,, L)$.

Check for crossing:

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**** Horizontal asymptotes are also known as "end behavior" since they describe what is happening at both ends of a function. ****

****A rational function will have a horizontal asymptote when the degree of the numerator is either less than or equal to the degree of the denominator.****

Think About It!

Sketch a possible graph for $\boldsymbol{f(x)}$ with the given set of conditions.

1. $\boldsymbol{f(0) = 4}$; $\boldsymbol{\lim_{x \to 0} f(x) = 2}$; No Zeros

$\boldsymbol{\lim_{x \to 1^-} f(x) = \infty}$; $\boldsymbol{\lim_{x \to 1} f(x) = DNE}$; $\boldsymbol{\lim_{x \to 5^+} f(x) = \infty}$; $\boldsymbol{\lim_{x \to 5^-} f(x) = -\infty}$;

$\boldsymbol{\lim_{x \to -\infty} f(x) = 1}$; $\boldsymbol{\lim_{x \to \infty} f(x) = 1}$; Cross point at $\boldsymbol{x = 7}$