

Definition of Continuity

A function f is said to be continuous at $x = c$ if and only if:

- 1) $f(c)$ is defined.
- 2) $\lim_{x \rightarrow c} f(x)$ exists.
- 3) $\lim_{x \rightarrow c} f(x) = f(c)$.

A Function is continuous on an open interval (a, b) if it is continuous at every point in (a, b) .

A **CONTINUOUS FUNCTION** is one that is continuous at every point of its domain.

Ex. Sketch a function f so that:

(a) $f(c)$ is not defined	(b) $\lim_{x \rightarrow c} f(x)$ does not exist
(c) $f(c)$ is defined and $\lim_{x \rightarrow c} f(x)$ exists but $\lim_{x \rightarrow c} f(x) \neq f(c)$	(d) f is continuous at $x = c$

Types of Discontinuities

Removable	Non-Removable		

A function f is continuous at a left endpoint a or a right endpoint b if

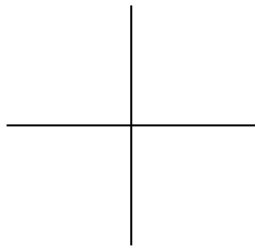
$$\lim_{x \rightarrow a^-} f(x) = f(a) \text{ or } \lim_{x \rightarrow b^+} f(x) = f(b).$$

Ex. The function $f(x) = \sqrt{25 - x^2}$ is continuous on the closed interval $-5 \leq x \leq 5$ because

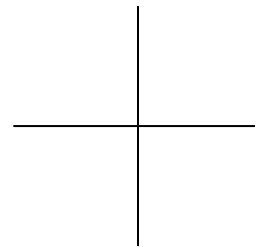
$$\lim_{x \rightarrow -5^+} f(x) = f(-5) = 0 \text{ and } \lim_{x \rightarrow 5^-} f(x) = f(5) = 0.$$

- Ex A.** 1) Find the value(s) of x at which the given function is discontinuous.
 2) Identify each value as a point, jump, or infinite (asymptotic) discontinuity
 3) Identify each value as a removable or non-removable discontinuity.
 4) Sketch the graph.

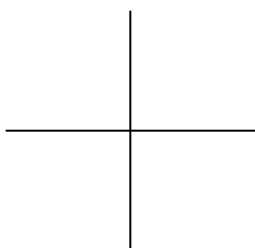
(a) $f(x) = \frac{x^2 - 4}{x - 2}$



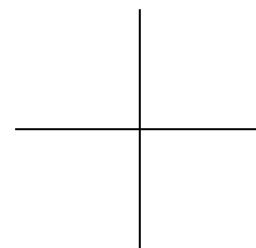
(b) $f(x) = \frac{1}{x - 3}$



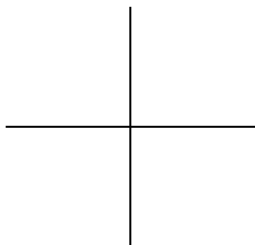
(c) $f(x) = \frac{|x - 3|}{x - 3}$



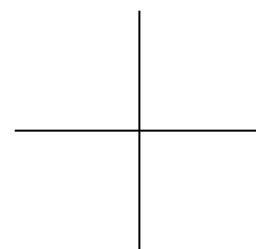
(d) $f(x) = \begin{cases} x + 2 & \text{if } x < 1 \\ 2 - x & \text{if } x > 1 \end{cases}$



(e) $f(x) = x^2 + 1$



(f) $f(x) = \frac{x}{x^2 - 3x}$



- Ex B.** Use the definition of continuity to prove that the function is discontinuous at $x = 2$ if $a = -4$.
 Sketch the graph of f to check your answer.

$$f(x) = \begin{cases} ax + 7 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

