Definition of Continuity
A function f is said to be continuous at $x = c$ if and only if:
1) $f(c)$ is defined.
2) $\lim_{x \to c} f(x)$ exists.
3) $\lim_{x \to c} f(x) = f(c)$.

A Function is continuous on an open interval (a , b) if it is continuous at every point in (a , b).

A **CONTINUOUS FUNCTION** is one that is continuous at every point of its domain.

Ex. Sketch a function f so that:

(a) $f(c)$ is not defined	(b) $\lim_{x \to c} f(x)$ does not exist
(c) $f(c)$ is defined and $\lim_{x \to c} f(x)$ exists but $\lim_{x \to c} f(x) \neq f(c)$	(d) f is continuous at $x = c$

Types of Discontinuities

Removable	Non-Removable		

A function f is continuous at a left endpoint a or a right endpoint b if
$\lim_{x \to a^-} f(x) = f(a)$ or $\lim_{x \to b^+} f(x) = f(b)$.

Ex. The function $f(x) = \sqrt{25 - x^2}$ is continuous on the closed interval $-5 \le x \le 5$ because
$\lim_{x \to -5^+} f(x) = f(-5) = 0$ and $\lim_{x \to 5^-} f(x) = f(5) = 0$.

Ex A. 1) Find the value(s) of x at which the given function is discontinuous.

2) Identify each value as a point, jump, or infinite (asymptotic) discontinuity

3) Identify each value as a removable or non-removable discontinuity.

4) Sketch the graph.

(a) $f\left(x\right)=\dfrac{x^2-4}{x-2}$

(b) $f\left(x\right)=\dfrac{1}{x-3}$

(c) $f\left(x\right)=\dfrac{\left|x-3\right|}{x-3}$

(d) $f\left(x\right)=\begin{cases} x+2 \text{ if } x<1 \\ 2-x \text{ if } x>1 \end{cases}$

(e) $f\left(x\right)=x^2+1$

(f) $f\left(x\right)=\dfrac{x}{x^2-3x}$

Ex B. Use the definition of continuity to prove that the function is discontinuous at $x = 2$ if $a=-4$.
Sketch the graph of f to check your answer.

$$f\left(x\right)=\begin{cases} ax+7 \text{ if } x\neq 2 \\ \quad 1 \quad \text{ if } x=2 \end{cases}$$